THE ROLES OF SYSTEMATIC SKEWNESS AND SYSTEMATIC KURTOSIS IN ASSET PRICING

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DECLARATION

I, Minh Phuong Doan, declare that:

a. except where due acknowledgement has been made, this work is that of myself alone;

b. this work has not been submitted, in whole or part, to qualify for any other academic award;

c. the content of the thesis is the result of work that has been carried out since the official commencement date of the approved research program;

d. Ms Annie Ryan was paid for proofreading this work for grammar and clarity.

Signed Date 10/03/2011

Minh Phuong Doan

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ABSTRACT

The role of higher moments of a return distribution has become increasingly important in the literature mainly because traditional measures of risk based on the mean-variance framework have failed to fully characterise return behaviour (Samuelson 1970; Campbell and Hentschel 1992; Kirchler and Huber 2007). Given that the empirical stock return distribution is observed to be both asymmetric and leptokurtic, this study examines the importance of skewness and kurtosis in asset pricing in the context of the Australian market from 1992 to 2009.

The study focuses on systematic measures of skewness and kurtosis to investigate the importance of skewness and kurtosis in asset pricing. The study investigates skewness and kurtosis through three different aspects. Firstly, the study examines whether asset returns are mean-variance-skewness-kurtosis efficient rather than conventionally mean-variance efficient. Secondly, the study investigates whether skewness and kurtosis are important pricing factors for asset returns. Thirdly, the study examines whether systematic skewness and systematic kurtosis can effectively capture the market risk asymmetry between bull and bear markets.

To investigate the first aspect, the study conducts empirical tests of the capital asset pricing model (CAPM) within the multivariate linear regression framework. Within this framework, the study firmly rejects the hypothesis of mean-variance efficiency of asset returns and shows evidence that the market beta is not sufficient to explain asset returns. The study then investigates whether asset returns are mean-variance-skewness-kurtosis efficient. The analysis is carried out using a generalised multivariate method and a bootstrap method. The study finds that Australian asset returns are generally mean-variance-skewness-kurtosis efficient in the four

periods of 1992–1996, 1997–2001, 2002–2006 and 1992–2006. This provides convincing evidence that the four-moment model developed in this study has advantages in explaining patterns of asset returns over the two-moment model.

The second aspect is investigated using both time-series and cross-sectional approaches. The study uses the Fama and French (1992) methodology to examine time-series returns while the Fama and MacBeth (1973) two-pass methodology is used to examine asset returns in cross-section. The analysis using these procedures strongly suggests that systematic skewness and systematic kurtosis are important pricing factors for asset returns. Interestingly, the study finds that when systematic skewness and kurtosis are added to the CAPM model, they appear to be the dominant explanatory variables and make the market factor insignificant.

The final aspect is investigated in the context of the two-moment dual-beta model proposed by Bharadwai and Brooks (1993) and the four-moment dual-beta model developed in this study. Within the two-moment framework, the study finds that market risk asymmetry is significant in the Australian market. However, when systematic skewness and systematic kurtosis are added to the CAPM, these factors can capture the market risk asymmetry effectively. Finally, the study concludes that the four-moment model has advantages over the two-moment dual-beta model proposed by Bharadwai and Brooks (1993) because it provides an effective method to capture the asymmetry in risk factor loadings without having to specify bull and bear periods.

CHAPTER 1. INTRODUCTION

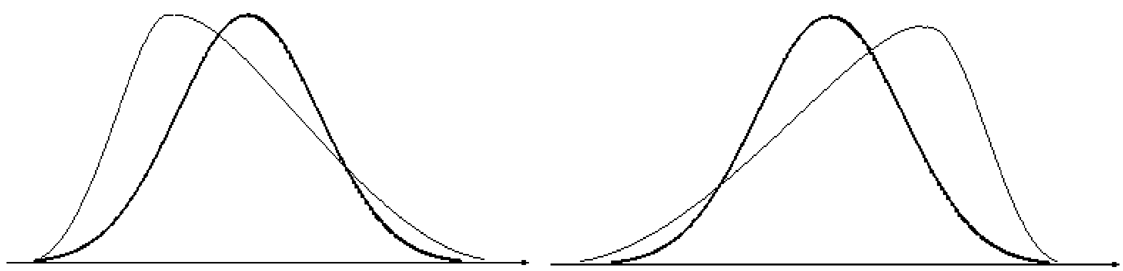
1.1 Understanding Skewness and Kurtosis

Skewness and kurtosis are statistical terms that, along with mean and standard deviation, help describe the overall shape of the probability distribution of a variable. Skewness is the third standardised moment of the probability distribution and it measures the lopsidedness or asymmetry of the distribution. Kurtosis is the fourth standardised moment of the probability distribution and it measures the heavy tails of the distribution.

A distribution with negative skewness has a longer tail in the lower-return side and a distribution with positive skewness has a longer tail on the higher-return side of the curve (figure 1.1). With a negatively skewed distribution, there is greater downside risk than what the standard deviation measures. Conversely, there is less downside risk than indicated by the standard deviation when the distribution is positively skewed. In other words, the standard deviation overstates the downside risk for a positively skewed distribution while understating the downside risk for a negatively skewed distribution.

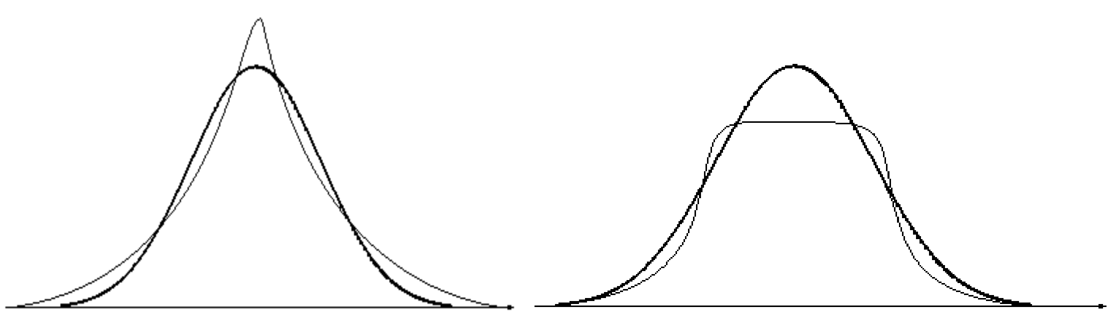
A distribution with kurtosis greater than 3 is a leptokurtic distribution where 3 is the kurtosis of a normal distribution. A leptokurtic distribution (figure 1.2) has a sharper peak and fatter tails compared to a normal distribution and it indicates a lower probability than a normally distributed variable of values near the mean and a higher probability than a normally distributed variable of extreme values. Conversely, a distribution with kurtosis less than 3 is a platykurtic distribution (figure 1.2). In term of shape, a platykurtic distribution has a lower, wider peak and thinner tails and it indicates a higher probability than a normally distributed variable of values near the mean and a lower probability than a normally distributed variable of extreme values.

Figure 1.1 Comparisons of the Normal Distribution to Skewed Distributions



Positive-skewed Distribution   Negative-skewed Distribution

Figure 1.2 Comparisons of the Normal Distribution to High Kurtosis and Low Kurtosis Distributions



Leptokurtic Distribution   Platykurtic Distribution

1.2 Causes of Skewness and Kurtosis in Asset Pricing

Damodaran (1985) was the first to point out that skewed distributions of asset returns are caused by investors reacting asymmetrically to good news and bad news from companies. Good news increases stock prices, yet some of this increase is diminished by the increase in the risk premium requested for the higher volatility. On the other hand, bad news lowers stock prices and this drop is amplified further by the increase in the risk premium requested for the higher volatility. This explains why return distributions are commonly negatively skewed.

Chen, Hong and Stein (2001) propose another reason for skewness. They argue that investor heterogeneity is central to this phenomenon. When differences of opinion among investors as to fundamental value are large, investors in the bear market, who are subject to short-sale constraints, are forced to sell all their shares and stay out of the market. Their prices may not fully reflect the information in the market. However, the sales of stocks due to short-sale constraints have sent a wrong signal to the market and caused stock prices to decrease significantly as a result of noise traders over-reacting to the current state of the market.

Karpoff (1997) uses the theory of transaction cost and recession-related premium to explain why the return premium is much higher in the downside risk than in the upside risk. He proposes that higher transaction costs in economic recessions contribute to higher return premium in the downturn. Schwert (1989) finds greater market volatility during recessions and this could contribute to greater bid-ask spreads which lead to higher traders' required premium to compensate for greater uncertainty.

The leptokurtic distribution, which is a common distribution of asset returns, is caused by volatility clustering (Campbell and Hentschel 1992). Kirchler and Huber (2007) propose that heterogeneity of fundamental information is the main driving force for trading activity, volatility and the emergence of fat tails. They also discovered that with respect to volatility clustering, the decrease of absolute returns after new information is released follows an intra-periodical pattern which yields a long-lasting positive autocorrelation of absolute returns. When information is released to the market, prices fluctuate greatly. This volatility reduces quickly as traders learn from past prices and react quickly to the information. This leads to relatively stable prices until new information is released again.

1.3 Measures of Skewness and Kurtosis

In the financial market, skewness and kurtosis of asset returns are measured as follows:

$$\text{Skewness} = \frac{1}{T-1}\sum_{t=1}^{T}\left[\frac{R_{it}-\bar{R}_i}{\sigma_i}\right]^3. \tag{1.1}$$

$$\text{Kurtosis} = \frac{1}{T-1}\sum_{t=1}^{T}\left[\frac{R_{it}-\bar{R}_i}{\sigma_i}\right]^4 - 3, \tag{1.2}$$

where R_i, $\bar{R}_i$ and σ_i are the returns, the expected return and the standard deviation of asset i respectively.

As skewness and kurtosis measures in equations (1.1) and (1.2) do not consider a market context, they are not useful in asset pricing and performance valuation. Kraus and

Litzenberger (1976) propose that systematic skewness, rather than total skewness, is relevant to market valuation. Systematic skewness is defined as the component of an asset's skewness that is related to the market portfolio's skewness. In this context, systematic skewness is considered a non-diversifiable measure of skewness and therefore it is consistent with the assumption of portfolio theory that only systematic risk is relevant to an investor's decision. Kraus and Litzenberger (1976) define the systematic measure of skewness as an analog to the market beta as the following:

$$S_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^2]}{\{R_m - E(R_m)\}^3}, \tag{1.3}$$

where R_i and R_m are the return of asset i and the market return respectively.

Unlike Kraus and Litzenberger (1976), Harvey and Siddique (2000) analyse the ability of conditional skewness (coskewness) to explain the cross-sectional variation of asset returns in comparison to other well-known risk factors. The coskewness compares the asset returns to the market returns, i.e. whether the asset's returns are more (positively) or less (negatively) skewed than the market's return.

The coskewness is defined as:

$$\gamma_i = \frac{E[\varepsilon_{i,t+1}\varepsilon^2_{M,t+1}]}{\sqrt{E[\varepsilon^2_{i,t+1}]}E[\varepsilon^2_{M,t+1}]}, \tag{1.4}$$

where $\varepsilon_{i,t+1} = r_{i,t+1} - \alpha_i - \beta_i r_{M,t+1}$ and $\varepsilon_{M,t+1} = r_{M,t+1} - E[r_M]$ where α_i and β_i are the regression estimates of the capital asset pricing model (CAPM) ; $r_{i,t+1}$, $r_{M,t+1}$ and $E[r_M]$ are the return of asset i at time t+1, the market return at time t+1 and the expected market return respectively.

As this study focuses on unconditional asset pricing models as well as testing efficiency of the models which include the market beta, it is more appropriate to construct systematic measures of skewness and kurtosis using the Kraus and Litzenberger (1976) approach. Using the methodology suggested by Kraus and Litzenberger (1976), this study defines systematic measure of kurtosis as:

$$K_i = \frac{E\left[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^3\right]}{\{R_m - E(R_m)\}^4}, \tag{1.5}$$

where R_i and R_m are the return of asset i and the market return respectively.

1.4 Problems Investigated

Since the pioneering work of Markowitz (1952), Sharpe (1964) and Lintner (1965), a growing literature documents the inefficiency of the CAPM. For example, Roll (1977) and Ross (1977) find evidence that the portfolio used as a market proxy is inefficient. Roll and Ross (1994) and Kandel and Stambaugh (1995) argue that even very small deviations from efficiency can produce an insignificant relationship between risk and expected returns. Campbell and Hentschel (1992) propose that volatility clustering, which is caused by large stock returns being followed by stock returns of similar magnitude but in the opposite direction, leads to fat-tail distributions. This volatility clustering effect is related to how information arrives and is received by the market. The clustering in return volatility has raised a fundamental question as to whether the asset pricing model based on the mean-variance framework is adequate in capturing variation in average stock returns.

It has been observed that asset returns are more highly correlated when below the mean (i.e. in a bear market) than when above the mean (i.e. in a bull market). Investors treat downside losses and upside gains asymmetrically, giving the former much heavier weight in their decisions than the latter, which is consistent with Kahneman and Tversky's loss aversion preferences (1979). Therefore, investors demand a higher premium for holding stocks with high downside risk. However, most capital asset pricing models fail to capture this variation in risk factor loadings as they suggest that the expected return is linearly proportional to the market premium in both bull and bear markets. The Inter-temporal CAPM by Merton (1973), the Arbitrage Pricing Theory (APT) by Roll (1977) and the Fama and French three-factor model (1992) improve the CAPM by including more explanatory variables rather than by incorporating asymmetry in risk factor loadings across up markets and down markets. Thus, the variation of risk factor loadings due to market conditions is still left unexplained.

1.5 Motivation of the Study

Given that the empirical stock return distribution is observed to be asymmetric and leptokurtic, a natural extension of two-moment models is to incorporate skewness and kurtosis. However, even for studies on the U.S. market, direct examination of skewness and kurtosis is quite limited and approaches used to examine them vary. For example, Kraus and Litzenberger (1976) expand the investor's utility function beyond the second moment in a Taylor series expansion to examine the skewness effect. Harvey and Siddque (2000) use conditional skewness to test whether stocks with large negative skewness can earn a high risk premium. Moreover, although asset returns are known to be both asymmetric and fat-tailed, numerous researchers focus only on the first three moments of the return distribution while neglecting kurtosis. However, kurtosis is also important because extreme returns occur too often to be consistent with normality. Based on these arguments, this study will explain how skewness and kurtosis might be relevant to the investor's decision.

While the variation of market risk in different market conditions is documented, are there any links between risk asymmetry and skewness and kurtosis factors? One clue that pushes this study in the direction of higher moments to explain risk asymmetry is that skewness and kurtosis are associated with non-normalities in the return distribution and therefore there is a possibility that downside and upside market risk are strongly correlated with skewness and kurtosis. In fact, Harvey and Siddique (2000) provide evidence that skewness captures some asymmetry in risk. The goal of this study is to go one step further and examine whether both skewness and kurtosis capture risk asymmetry effectively. The study includes both skewness and kurtosis because recent studies by McNeil and Frey (2000), Bali (2003) and Cotter (2004) document that measures of the market risk are largely influenced by extreme market returns and therefore by kurtosis risk; and negative extreme returns occur more often than positive extreme returns (i.e. skewness risk).

1.6 Contribution of the Study

This study will contribute to the current literature in the following ways. Firstly, this is the first Australian study to directly examine the roles of both skewness and kurtosis in asset pricing. Approaches to the examination of the importance of skewness and kurtosis in this study are different from those of the U.S. studies. While U.S. studies often use cross-sectional analysis to show the relevance of skewness and kurtosis to asset pricing, this study uses a combination of multivariate linear regressions and bootstrap methods to demonstrate that asset returns are mean-variance-skewness-kurtosis efficient rather than mean-variance efficient. The bootstrap method used in this study provides advantages in dealing with non-normal errors which previous studies usually ignore. The study also uses both time-series and cross-sectional regression approaches to examine whether skewness and kurtosis are pricing factors for asset returns.

Secondly, although the topic of varying beta risk has been widely documented, research studies only focus on conditional time-varying beta and therefore they do not show how beta instability affects the risk premium, particularly in bull and bear markets. To the author's knowledge, this is the first study in the literature to investigate the link between market risk asymmetry and skewness and kurtosis. In particular, the study investigates whether skewness and kurtosis can capture beta asymmetry in risk factor loadings effectively. If so, a capital asset pricing model incorporating skewness and kurtosis would capture asymmetry in risk factor loadings and therefore would have advantages over time-varying beta asset pricing models as proposed in the existing literature.

1.7 Research Questions

The purpose of this study is to explain the deficiencies of the CAPM and how skewness and kurtosis become relevant to asset pricing models in regard to bridging the gap between theory and empirical evidence. Research questions are as follows:

Research question 1

Are asset returns mean-variance efficient?

Research question 2

If asset returns are not mean-variance efficient, are they mean-variance-skewness-kurtosis efficient?

Research question 3

Are systematic skewness and systematic kurtosis pricing factors for asset returns?

Research question 4

Is market risk asymmetric between bull and bear markets? If it is, can systematic skewness and systematic kurtosis proxy for risk asymmetry caused by changes in market conditions?

1.8 Methodology and Main Findings

To answer the first research question, the study conducts empirical tests of the CAPM within a multivariate linear regression framework. In particular, the study employs the Wald test and the Gibbons, Ross and Shanken (GRS) test to investigate whether asset returns are mean-variance efficient. As the Wald and GRS tests for mean-variance efficiency are based on the assumption that the error term of the CAPM is multivariate normal, the study proposes a bootstrap test as a robustness test for conclusions drawn from the Wald and GRS tests. Using Australian data from 1992 to 2009, the study firmly rejects the hypothesis of mean-variance efficiency of asset returns. The study also shows clear evidence that the market beta alone is not sufficient to explain asset returns. The finding is robust when bootstrapping is used to overcome problems associated with non-normalities in the error term. As asset returns are not mean-variance efficient while skewness and kurtosis are associated with non-normalities in asset returns, this leads the study to investigate whether asset returns are mean-variance-skewness-kurtosis efficient.

The study develops a four-moment model by incorporating systematic skewness and systematic kurtosis in the CAPM to address the second research question. The study uses a generalised multivariate method proposed by Gibbons, Ross and Shanken (1989) to investigate whether the four-moment model can effectively explain patterns of asset returns. This is equivalent to investigating whether asset returns are mean-variance-skewness-kurtosis efficient. The study finds that the four-moment model adequately explains variation in asset returns in the four periods of 1992–1996, 1997–2001, 2002–2006 and 1992–2006, but not in the 2007–2009 period of the global financial crisis. Conclusions drawn from the generalised GRS test are found to be robust when bootstrapping is used as a robustness test.

To address the third research question, the study uses the two-pass cross-sectional methodology proposed by Fama and MacBeth (1973). The cross-sectional analysis using this procedure shows that systematic skewness and systematic kurtosis do command significant risk premiums. Interestingly, the study finds that when systematic skewness and kurtosis are added to the CAPM model, they are the dominant explanatory variables and make the market factor insignificant. As the Fama and MacBeth (1973) two-pass cross-sectional methodology has been criticised for the errors-in-variables problem in the second pass estimation, the study uses the Shanken (1992) approach and the higher moment estimators approach proposed by Dagenais and Dagenais (1997) to correct the problem. The results from these two approaches suggest that the errors-in-variables problem causes the significance of the market premium, of the systematic skewness premium and of the systematic kurtosis premium measured by the traditional cross-sectional regressions to be overstated. Nevertheless, the results show that systematic skewness and systematic kurtosis retain their significance as pricing factors for asset returns after the error-in-variables problem is corrected.

The final research question is addressed by a time-series analysis using the two-moment dual-beta model proposed by Bharadwai and Brooks (1993) and the four-moment dual-beta models developed in this study. Using the two-moment dual-beta model proposed by Bharadwai and Brooks (1993), the study finds that the downside risk and the upside risk are priced asymmetrically and the return premium for the downside risk is significantly higher than the premium for the upside risk. Using the four-moment dual-beta model, the study finds that systematic skewness and systematic kurtosis capture the asymmetry in risk factor loadings effectively and so proxy for the risk asymmetry caused by investors reacting asymmetrically between bull and bear markets. Finally, the study concludes that the four-moment model has advantages over the two-moment dual-beta model proposed by Bharadwai and Brooks (1993) because it provides an effective method to capture the asymmetry in risk factor loadings without specifying bull and bear market conditions.

1.9 Structure of the Thesis

Chapter 2 surveys the relevant literature of mean-variance efficiency tests and previous studies of higher moments in asset pricing. Chapter 3 gives an overview of the Australian data from 1992 to 2009 and provides descriptive evidence about the relevance of systematic skewness and systematic kurtosis to asset pricing. Chapter 4 is devoted to mean-variance efficiency tests to address the first research question. Chapter 5 presents mean-variance-skewness-kurtosis efficiency tests to address the second research question. The third research question is dealt with in Chapter 6, which addresses the issue of whether systematic skewness and systematic kurtosis are pricing factors for asset returns. Analysis of the fourth research question is presented in Chapter 7, where market risk asymmetry using dual-beta models is the focus of attention. The advantages of using the four-moment model are highlighted in the chapter as it appears that skewness and kurtosis capture beta asymmetry associated with bull and bear markets. Conclusions and final discussion are provided in Chapter 8.

CHAPTER 2. LITERATURE REVIEW

2.1 Introduction

The foundation of the portfolio theory and the CAPM by Markowitz (1952), Sharpe (1964) and Lintner (1965) have led to numerous studies in asset allocation based upon the first two moments of the return distribution. There are two main reasons for the popularity of the CAPM in asset allocation. Firstly, the model is intuitively appealing as it provides a simple performance comparison for alternative assets because their risk and expected return characteristics can be presented on a two-dimensional graph. Secondly, the mean-variance approach assumes Gaussian distributions for asset returns and therefore the model is fully compatible with expected utility maximisation regardless of investors' preferences (Berk 1997 and Sentana 2009).

In the mean-variance framework, asset returns are assumed to be normally distributed. However, several empirical tests on the Sharpe's CAPM (1964) have largely rejected the validity of the model which assumes that an investor's utility function is quadratic and that co-movement with the market return is the only important factor in pricing assets. For example, Roll (1977) and Ross (1977) find evidence that the portfolio used as a market proxy is inefficient. Roll and Ross (1994) and Kandel and Stambaugh (1995) argue that even very small deviations from efficiency can produce an insignificant relationship between risk and expected returns. Samuelson (1970) and Rubinstein (1973) argue that higher moments are relevant to the investor's decision because asset returns are driven by asymmetric fat-tailed distributions and extreme returns occur too often to be consistent with normality.

The purpose of this chapter is to review the theoretical and empirical studies which attempt to explain the deficiencies of the CAPM and how skewness and kurtosis are relevant to asset pricing models in regard to bridging the gap between theory and empirical evidence. Section 2.2 reviews studies on mean-variance efficiency tests of the CAPM. Section 2.3 addresses the relevance of skewness and kurtosis to asset pricing and considers how this study can contribute to the current literature. Section 2.4 concludes the chapter.

2.2 Mean-Variance Efficiency Tests

A portfolio is defined as mean-variance efficient with respect to a given set of assets in the portfolio if it is not possible to form another portfolio of those assets with the same expected return but a lower variance, or with the same variance but a higher expected return. Despite the simplicity of this definition, testing for mean-variance efficiency is important in many practical situations, such as mutual fund performance, value gained from portfolio diversification and tests of linear factor models (including the CAPM and the APT), which imply that dependent portfolios must be mean-variance efficient (De Roon and Nijman 2001; Errunza *et al.* 1999).

Since the work of Gibbons (1982), empirical tests of the CAPM are usually conducted within a multivariate linear regression (MLR) framework. Statistical inference for the MLR model in econometrics and empirical finance is traditionally based either on asymptotic approximation or on finite-sample distribution theory. The Wald test is one of the most popular tests relying on the asymptotic approximation method while the Gibbons, Ross, Shanken (GRS) (1989) test is the well-accepted test of mean-variance efficiency using the finite-sample distribution approach. Together with the Wald and GRS tests, the bootstrap method is used to assess the robustness of inferences drawn from the Wald and the GRS tests. Details of these methods are described in the following sections.

2.2.1 Wald Test

Define r_t as an $(N \times 1)$ vector of excess returns of N assets (or portfolios of assets). For these N assets, the excess returns can be described using the CAPM:

$$r_t = \alpha + \beta r_{mt} + \varepsilon_t; \tag{2.1}$$

where $\varepsilon_t \sim N(0, \Sigma)$ which satisfies:

$$E[\varepsilon_t] = 0, \tag{2.2}$$

$$E[\varepsilon_t \varepsilon_t'] = \Sigma, \tag{2.3}$$

$$E[r_{mt}] = \mu_m, \tag{2.4}$$

$$E[(r_{mt} - \mu_m)^2] = \sigma^2 \text{ and} \tag{2.5}$$

$$Cov[r_{mt}, \varepsilon_t] = 0, \tag{2.6}$$

β is the $(N \times 1)$ vector of betas, r_{mt} is the market portfolio excess return at period t, α is the $(N \times 1)$ vector of intercepts and ε_t is the $(N \times 1)$ vector of disturbances at time t.

The implication of the CAPM is that all of elements of vector α are zero, that is there are no abnormal returns on average. Therefore, a standard approach to test the efficiency of the CAPM is to test the null hypothesis:

$$H_0: \alpha = 0, \tag{2.7}$$

or that all the intercepts are zero, against the alternative hypothesis:

$$H_1 : \alpha \neq 0, \tag{2.8}$$

that at least one of the intercepts is non-zero.

The estimates of the CAPM are generated using the maximum likelihood approach with:

$$\hat{\alpha} = \hat{\mu} - \hat{\beta}\hat{\mu}_m \ , \tag{2.9}$$

$$\hat{\beta} = \frac{\sum_1^T (r_t - \hat{\mu})(r_{mt} - \hat{\mu}_m)}{\sum_1^T (r_{mt} - \hat{\mu}_m)^2} \quad \text{and} \tag{2.10}$$

$$\hat{\Sigma} = \frac{1}{T}\sum_{t=1}^{T}\left(r_t - \hat{\alpha} - \hat{\beta}r_{mt}\right)\left(r_t - \hat{\alpha} - \hat{\beta}r_{mt}\right)', \tag{2.11}$$

where $\hat{\mu} = \frac{1}{T}\sum_1^T r_t$ and $\hat{\mu}_m = \frac{1}{T}\sum_1^T r_{mt}$.

It is assumed that r_{m1}, r_{m2},.., r_{mT} are i.i.d and follow multivariate normal distributions.

The Wald-statistic for the null hypothesis is calculated as follows:

$$W = \hat{\alpha}'[\mathrm{Var}(\hat{\alpha})]^{-1}\hat{\alpha}$$

$$= h^{-1}\, \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\ , \tag{2.12}$$

where $h = \frac{1}{T}(1 + \hat{\theta}^2)$; $\hat{\theta} = {}^{\mu_m}/_{\sigma_m}$; $\bar{r}_m$ is the sample mean of r_{mt} and s is the sample standard deviation of r_{mt}. Under the null hypothesis, W has a chi-square distribution with N degrees of freedom.

2.2.2 Gibbons, Ross and Shanken (1989)Test of Mean-Variance Efficiency

Several studies in econometrics and finance literature (MacKinlay 1987; Shanken 1996; Campbell *et al.* 2001; Dufour and Khalaf 2002) have shown that standard asymptotic theory provides a poor approximation to large-sample and finite-sample distributions of MLR-based tests. In particular, distortions of size increase quickly as the number of equations increases. As a result, conclusions drawn from empirical studies of the CAPM using the MLR method could lead to a spurious rejection of the null hypothesis.

In this context, as emphasised by Shanken (1996) and Campbell *et al.* (2001), statistical methods applied to a finite sample appear to be important. As a result, a number of studies (Jobson and Korkie 1982; MacKinlay 1987; Gibbons, Ross and Shanken (GRS) 1989; Zhou 1991; Stewart 1997) propose tests based on finite-sample distributional theory in order to test the CAPM. These tests exploit results from multivariate analysis of variance, in particular Hotelling's T^2 statistic, which is a multivariate generalisation of the univariate t-statistic. Similar to the Wald test, these studies are based on the assumption that the errors in the CAPM follow a Gaussian distribution. This may provide valid tests for the Gaussian CAPM but not for the alternative non-Gaussian CAPM.

Among several studies of mean-variance efficiency using the finite-distribution approach, the GRS (1989) study appears to perform well. Affleck-Graves and McDonald (1989) provide evidence that the GRS test is reasonably robust to minor departures from normality. In this study, several empirical analyses using multivariate linear regressions rely on the GRS method. Therefore, it is important to understand how the GRS test works. First, the study considers the following multivariate linear regression model in a general form:

$$\tilde{r}_{it} = \alpha_{0i} + \sum_{j=1}^{L} \beta_{ij} \tilde{f}_{jt} + \tilde{\varepsilon}_{it} \quad \forall i = 1, .., N \text{ and } \forall\, t = 1, .., T \ , \qquad (2.13)$$

where $\tilde{r}_{it}$ is the excess return on asset i in period t; $\tilde{f}_{jt}$ is the explanatory factor j in period t, $\tilde{\varepsilon}_{it}$ is the disturbance term for asset i in period t, and is assumed to be normally distributed. $\tilde{\varepsilon}_{it}$ needs to satisfy the following conditions:

$$E[\tilde{\varepsilon}_{it}] = 0, \qquad (2.14)$$

$$E\{\tilde{\varepsilon}_{it}\tilde{\varepsilon}_{it}{}'] = \Sigma \text{ and} \qquad (2.15)$$

$$Cov[\tilde{f}_{jt}\tilde{\varepsilon}_{it}] = 0 \quad . \qquad (2.16)$$

If the multivariate model in equation (2.13) is estimated using the ordinary least squares (OLS) method for each individual equation, the estimated intercepts have a multivariate normal distribution conditional on $\tilde{r}_{it}$, with:

$$\sqrt{T/\left(1+\hat{\theta}^2\right)}\ \hat{\alpha}_0 \ \sim N\left\{\sqrt{T/\left(1+\hat{\theta}^2\right)}\alpha_0 ; \Sigma\right\}, \qquad (2.17)$$

where $\hat{\alpha}_0 = (\hat{\alpha}_{01}, \hat{\alpha}_{02}, \ldots, \hat{\alpha}_{0N})$; $\hat{\theta} = \bar{f}/_S$ where $\bar{f}$ is the vector of sample mean of $\tilde{f}_{jt}$ and s^2 is the vector of sample variance of $\tilde{f}_{jt}$.

The hypothesis of mean-variance efficiency is equivalent to the hypothesis of multivariate zero intercepts as follows:

$$H_0\colon \alpha_{0i} = 0 \qquad \forall\, i = 1, \ldots, N. \tag{2.18}$$

To test the hypothesis, the GRS-statistic is computed as:

$$\text{GRS} - \text{statistic} = \left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left(1 + \bar{f}'\hat{\Omega}^{-1}\bar{f}\right)^{-1}\left(\hat{\alpha}_0{}'\hat{\Sigma}^{-1}\hat{\alpha}_0\right), \tag{2.19}$$

where $\bar{f}$ is the vector of sample mean of $\tilde{f}_{jt} = (\tilde{f}_{1t}, \tilde{f}_{2t} \ldots, \tilde{f}_{Lt})$;

$\hat{\Omega}$ is the sample variance-covariance matrix of $\tilde{f}_{jt}$;

$\hat{\Sigma}$ is the sample variance-covariance matrix of the errors; and

$\hat{\alpha}_0 = (\alpha_{01}, \alpha_{02}, \ldots, \alpha_{0N})$ is the vector of the intercepts where α_{0i} is the intercept of the regression of portfolio i on L regression parameters;

N is the number of underlying assets;

L is the number of regression parameters; and

T is the number of time-series observations.

The multivariate asset pricing test proposed by GRS (1989) has received considerable attention in the field because of its broad application. Although Affleck-Graves and McDonald (1989) argue that the GRS (1989) test is reasonably robust to departures from normality, it would be useful to have finite-sample tests that allow for non-Gaussian errors. However, there is little research on the effects of violating the multivariate normality assumption in the context of the GRS test. Since financial data does not always satisfy the above conditions, it is worth investigating the effects of non-Gaussian errors on the results of mean-variance efficiency tests.

2.2.3 Using Bootstrap Method for Mean-Variance Efficiency Test in the Case of Multivariate Non-normal Errors

The assumption that the error term is normal is critical in statistical analysis, for example, the Wald and GRS tests for mean-variance efficiency. However, this assumption may not always hold and a statistical method relying on this assumption may fail to give satisfactory results because standard deviations generated from traditional parametric methods would generate standard statistics that are invalid for statistical inference. In this case, the bootstrap method that relaxes this assumption may provide more reliable results.

Bootstrapping is a nonparametric approach that relies on the assumption that the current sample represents the population and therefore the empirical distribution function is a nonparametric estimate of the population distribution (Guan 2003). From the sample dataset, the desired statistic can be calculated as an empirical estimate of the true parameter. This approach uses the same theory underlying Monte Carlo simulation methods, except that it resamples from the original data rather than from an assumed population. When the sample size is large, the bootstrapped estimates converge to the true parameters as the number of repetitions increases. As a result, bootstrapped standard errors may tend to be more conservative than parametric estimates and therefore cover a wider range for the estimated coefficients. To measure the precision of the estimates, bootstrapped standard errors are calculated based on three steps:

1. Draw random samples with replacement from the sample dataset.
2. Estimate the desired statistic for each bootstrap sample, to determine the sampling distribution of the desired statistic.
3. Calculate the statistics of interest using the sampling distribution.

Although the bootstrap method can reduce the problems of non-normality, there are few applications of the bootstrap method in analysing the effects of non-normality in mean-variance efficiency tests. The main contributions are from Affleck-Graves and McDonald (1989), Zhou (1993) and Chou and Zhou (2006). Affleck-Graves and McDonald (1989) test the robustness of the GRS test to the non-normality in the residual covariance matrix by using simulation techniques. They conclude that the multivariate GRS test is robust with respect to typical levels of non-normality. Zhou (1993) reconsiders the GRS problem for elliptical distributions and notes some invariance properties of the Hotelling statistic for testing mean-variance efficiency. He also proposes the use of a simulation procedure to approximate the p-values for the GRS-statistic. Chou and Zhou (2006) test the well-known Fama and French (1993) method using the bootstrap method when the error distributions are not specified.

Despite its popularity, mean-variance analysis neglects the effect of higher-order moments on asset allocation and asset pricing. In particular, it ignores the third and fourth central moments of returns, which are skewness and kurtosis respectively. Higher moments are crucial in analysing derivative assets, games of chance and insurance contracts. Adding to the current literature, this study investigates the roles of higher moments, in particular systematic skewness and systematic kurtosis, in pricing factors for asset returns. The study in the subsequent chapters tests the hypothesis of mean-variance-skewness-kurtosis efficiency using applications of the generalised GRS test and the bootstrap method, as mentioned previously.

2.3 The Relevance of Skewness and Kurtosis in Asset Pricing

It has been well documented that asset returns are driven by asymmetric distributions and that extreme returns occur too often to be consistent with a normal distribution. For example, Samuelson (1970) and Rubinstein (1973) argue that higher moments are relevant to the investor's decision unless the asset returns are normally distributed and the investor's utility functions are quadratic. Several empirical tests based on Sharpe's CAPM (1965) have largely rejected the CAPM in favor of the higher-moments models. Details of these studies are presented as follows.

2.3.1 Relationships between Beta Asymmetry and Higher Moments

Higher-order moment models emerge from numerous empirical studies. For example, the studies of Fabozzi and Francis (1977, 1979) and Kim and Zumwalt (1979) argue that the parameter estimates of the CAPM are asymmetric between upside and downside markets and therefore total variation of asset returns should be divided into two parts: the variations when the market is up and when it is down. Investors are risk-averse and they treat downside losses and upside gains asymmetrically, giving the former much heavier weight in their decisions than the latter. This implies that investors' preferences are positively related to the size of variance in the up market and negatively related to the size of variance in the downside. These studies argue that variation in the market risk factor due to changes in market conditions is unexplained by the Sharpe-Litner CAPM.

Pettengill, Sundaram and Mathur (1995) argue that the risk-return relationship explained by the Sharpe-Litner CAPM is biased against finding a significant relationship between the market beta and expected returns. This is because the relationship between the beta and asset returns is conditional on market returns and therefore is conditional on market conditions. In the upside market, high beta securities should be rewarded for bearing risk with higher returns than low beta securities, but in the downside market the high-beta securities should experience lower returns than the low-beta securities. Thus, standard tests are biased against finding a relationship between the beta and the asset returns because these tests mix periods when the relationship between the beta and the asset returns is positive in the upside market with periods when this relationship is negative in the downside market. The study of Pettengill, Sundaram and Mathur (1995) proposes that if there is no systematic relationship between asset returns and the market beta, continued reliance on the beta as a measure of risk is inappropriate.

To solve the problem of beta asymmetry, Bhardwaj and Brooks (1993) developed the idea of varying systematic risk over bull and bear markets and established an asset-pricing model with time-varying risk. They use dummy variables to generate simultaneous bull and bear market estimates of alpha and beta. Their study classifies periods as either bull or bear markets by determining whether the market return is higher or lower than the mean market return for each period they examine. They test the validity of their classification of the market to determine if the process effectively identifies the presence of a rising or a falling market. Using U.S. data from the National Bureau of Economic Research (NBER) to describe economic peaks and troughs, their study finds a strong relationship between their classification of bull and bear markets with the presence of business cycle highs and lows. Their empirical results confirm a significant direct relationship between the market beta and the expected returns in the upside market and a significant inverse relationship between the market beta and the returns in the downside market.

While the importance of market risk asymmetry to asset pricing is widely documented, there is little work focusing on the connection between beta asymmetry and skewness and kurtosis factors. As skewness and kurtosis are associated with non-normalities in asset returns, there is a possibility that the downside and upside betas are correlated with skewness and kurtosis. Recent studies document the asymptotic distribution of extreme returns where negative extreme returns occur more often than positive extreme returns (McNeil and Frey 2000; Bali 2003; Cotter 2004). Since the literature exploring the relationship between beta asymmetry and skewness and kurtosis is limited, this study contributes to the literature through its focus on whether skewness and kurtosis proxy for the risk asymmetry associated with the changes in market conditions.

2.3.2 Asset Pricing Model in the Four-moment Framework

It is observed that asset return distributions are often skewed and exhibit fat tails. The role of higher moments has become increasingly important in the literature mainly because traditional measures of risk based on the mean-variance framework have failed to fully characterise variation in asset returns. This leads researchers to investigate the roles of higher moments in asset pricing.

2.3.2.1 How Skewness Enters Asset Pricing

One of the most important and frequently cited studies in the field is that of Kraus and Litzenberger (1976). This was the first study to fully establish a general higher-moment framework for asset pricing. Before Kraus and Litzenberger (1976), several studies (Alderfer and Bierman 1970; Peterson 1969; Samuelson 1970; Tsaing 1972) considered moments higher than variance but not in the market context. Arditti (1967) and Rubinstein (1973) considered skewness in a market context but did not distinguish between systematic and conditional skewness. Kraus and Litzenberger (1976) proposed that systematic skewness, rather than total skewness, is relevant to market valuation.

According to Kraus and Litzenberger (1976), the market portfolio is not mean-variance efficient but it is efficient with respect to investors' utility functions. Therefore, their basic approach is to expand a utility function beyond the second moment in a Taylor series to examine skewness effects. According to their research, if the rate of return on the market portfolio is non-symmetrically distributed, stock returns follow the model specified as follows:

$$\bar{r}_i = \alpha_i + b_1\beta_i + b_2\gamma_i + u_i, \tag{2.20}$$

where:

$$\beta_i = \frac{E[\{R_{it} - E(R_i)\}\{R_{mt} - E(R_m)\}]}{\{R_{mt} - E(R_m)\}^2}, \tag{2.21}$$

$$\gamma_i = \frac{E\left[\{R_{it} - E(R_i)\}\{R_{mt} - E(R_m)\}^2\right]}{\{R_{mt} - E(R_m)\}^3}, \tag{2.22}$$

where β_i is the market beta, γ_i is the systematic skewness of asset i, R_{it} is the return of asset i and R_{mt} is the return of the market index at time t. Using monthly NYSE data from 1935 to 1970, Kraus and Litzenberger (1976) show that when the capital asset pricing model is extended to incorporate systematic skewness, the price of systematic skewness is significant and the intercept of the security market line in excess return is zero.

Unlike Kraus and Litzenberger (1976), Harvey and Siddique (2000) analyse the ability of conditional skewness to explain the cross-sectional variation of asset returns in comparison to other well-known risk factors. Conditional skewness compares the returns of the asset to the market returns, i.e. whether the asset's returns are more (positively) or less (negatively) skewed than the market's return. The study of Harvey and Siddique is motivated by the fact that the standard CAPM fails to explain the returns of specific assets or groups of assets such as the smallest market-capitalised deciles and returns from specific strategies such as ones based on momentum. These assets have the most skewed returns. Therefore, skewness may be important for investment decisions because of induced asymmetries in realised returns. Conditional skewness is defined as:

$$\gamma_i = \frac{E[\varepsilon_{i,t+1}\varepsilon^2_{M,t+1}]}{\sqrt{E[\varepsilon^2_{i,t+1}]}E[\varepsilon^2_{M,t+1}]}, \tag{2.23}$$

where $\varepsilon_{i,t+1} = r_{i,t+1} - \alpha_i - \beta_i r_{M,t+1}$, and $\varepsilon_{M,t+1} = r_{M,t+1} - E[r_M]$ where α_i and β_i are the regressor estimates of the CAPM ; $r_{i,t+1}$, $r_{M,t+1}$ and $E[r_M]$ are the return of asset i at time t+1, the market return at time t+1 and the expected market return respectively.

Harvey and Siddique (2000) find that conditional skewness can capture downside risk, which is important in the context of Value-at-Risk (VaR). They propose that pricing errors in portfolio returns using mean-variance asset pricing models can also be partly explained using conditional skewness. Furthermore, they find that the model incorporating conditional skewness is helpful in explaining beta asymmetry in cross-sectional variation of asset returns where previous studies have been unsuccessful.

The studies of Kraus and Litzenberger (1976) and Harvey and Siddique (2000) have provided evidence that systematic skewness and conditional skewness are important to asset pricing because they characterise the true distribution of asset returns. However, it is still unclear what economic mechanism causes skewness. Damodaran (1985) points out that negative skewness can result from the distribution of good and bad news from companies. Campbell and Hentschel (1992) test the idea of Damoran (1985) and argue that skewness is caused by investors reacting asymmetrically to good news and bad news. Good news increases stock prices, yet some of this increase is diminished by the increase in the risk premium requested for the higher volatility. On the other hand, bad news lowers stock prices and this drop is amplified further by the increase in the risk premium requested for the higher volatility.

Chen, Hong and Stein (2001) propose another reason for skewness. They argue that investor heterogeneity is central to this phenomenon. When differences of opinion among investors as to fundamental value are large, investors in the bear market, who are subject to short-sale constraints, are forced to sell all their shares and stay out of the market. Their prices may not fully reflect the information in the market. However, the sales of stocks due to short-sale constraints sends a wrong signal to the market and causes stock prices to decrease significantly as a result of noise traders over-reacting to the current state of the market. Overall, the Chen, Hong and Stein (2001) study proposes that negative skewness is most pronounced in stocks that:

(1) experience an increase in trading volume relative to trend over the prior six months;

(2) have positive returns over the prior 36 months; and

(3) are large in terms of market capitalisation.

Smith (2007) fits the conditional three-model factor proposed by Harvey and Siddique (2000a) which allows prices of risk factors to vary over time. He finds that adding the size (SMB) and the book to market factors (HML) of Fama and French (1993) has little impact on the price of market beta risk when conditional skewness is included in the model. This implies that part of the ability of SMB and HML to explain return variation is related to conditional skewness. In addition, he also finds that when conditional skewness is positive, investors are willing to trade off a larger return per unit of gamma (a standard measure of conditional skewness risk) while only demanding a smaller premium when conditional skewness is negative. Similarly, the conditional skewness factor proposed by Harvey and Siddque (2000) indicates that a larger return per unit of gamma is given for the trade-off when the market is positively skewed but only a smaller return premium is required for bearing gamma risk when the market is negatively skewed.

In contrast, Chung, Johnson and Shill (2007) argue that non-market factors such as SMB and HML are proxies for higher-order conditional moments (comoments) and given the complication of estimating higher-order comoments, the Fama and French factors could be superior in practice. However, their results lack robustness, especially given the possibility of errors-in-variables in their cross-sectional regressions. The findings are also contradictory to the study of Ajili (2004) which concludes that conditional skewness is important in explaining variation in stock returns in the French market and it is not subsumed by HML and SMB.

Although the debate about whether non-market factors such as HML, SMB and mometum can explain the skewness of the asset returns is still ongoing, the studies of Chung, Johnson and Shill (2007) and Ajili (2004) find that the existence of skewness cannot be fully explained by size, book to market and momentum factors. These studies also show that the impact of skewness on asset pricing can be weak or strong, depending on the market, the type of stock and the period examined. Finally, the studies argue that skewness effects are strong for downside risk but less apparent for upside risk.

2.3.2.2 The Importance of Kurtosis in Asset Pricing

Most researchers concentrate on the first three moments, i.e. within the mean-variance-skewness framework, while neglecting kurtosis. Samuelson (1970) proposes that kurtosis reflects the probability of extreme events. The studies of Mandelbrot (1963) and Mandelbrot and Taylor (1967) show that financial market returns are not Gaussian but exhibit fat tails. Campbell and Hentschel (1992) suggest that news generates volatility and clustering. Trading is slow when no information is available. As soon as information is released to the market, prices vary significantly. Therefore, the greater the kurtosis is, the greater the volatility and the greater the probability of extreme events happening. However, they also argue that although good news increases stock prices, some of this increase is diminished by the increase in risk premium required to take on greater higher volatility. On the other hand, bad news lowers stock prices and this drop is amplified further by the increase in the risk premium. Because of clustering in news, the left tail of the return distribution is thicker than the right tail.

Kirchler and Huber (2007) find a significant positive relationship between the degree of heterogeneity of fundamental information and absolute returns. Heterogeneity of fundamental information is the main driving force for trading activity, volatility and the emergence of fat tails. Kirchler and Huber also discover that, with respect to volatility clustering, the decrease in absolute returns after new information is released follows an intra-period pattern which yields a long-lasting positive autocorrelation in absolute returns. When information is released to the market, prices fluctuate greatly but this volatility reduces quickly as traders react quickly to the information. Once prices reflect the new information, they become more stable until new information is released again.

Hwang and Satchell (1999) extend the Kraus and Litzenberger three-factor model to incorporate kurtosis. They suppose that there is a riskless asset, R_f, and there are n risky assets of which i^{th} return is represented as R_i. They assume that for an investor, the initial investment is 1 and the end of the period of wealth is w where w is as follows:

$$w = x_0 (1 + r_f) + \sum_{i=1}^{N} x_i (1 + r_i), \tag{2.24}$$

where x_0 and x_i are the proportions of riskless asset and risky asset i respectively, which satisfy:

$$x_0 + \sum_{i=1}^{N} x_i = 1. \qquad (2.25)$$

The portfolio return is as follows:

$$r_p = x_0 r_f + \sum_{i=1}^{N} x_i r_i. \qquad (2.26)$$

Hwang and Satchell (1999) define the systematic measures of variance, skewness and kurtosis for asset i as β_i, γ_i and θ_i respectively. They develop a relationship between the end of period wealth and systematic risk measures of the portfolio as follows:

Standard deviation of the portfolio is:

$$\sigma(w) = \sum_{i=1}^{N} x_i \beta_i \sigma(r_m), \qquad (2.27)$$

where $\sigma(z) = E[\{z - E(z)\}^2]^{1/2}$.

Systematic skewness of the portfolio is:

$$\gamma(w) = \sum_{i=1}^{N} x_i \gamma_i \gamma(r_m), \qquad (2.29)$$

where $\gamma(z) = E[\{z - E(z)\}^3]^{1/3}$.

Systematic kurtosis of the portfolio is:

$$\theta(w) = \sum_{i=1}^{N} x_i \theta_i \theta(r_m), \qquad (2.30)$$

where $\theta(z) = E[\{z - E(z)\}^4]^{1/4}$.

Under four-moment criterion, expected utility is described as:

$$E[U(w)] = f(E(w), \sigma(w), \gamma(w), \theta(w)). \qquad (2.31)$$

By including higher moments, the objective of maximising expected utility aligns with the objective of optimising asset allocation.

2.4 Applications of Skewness and Kurtosis

As skewness and kurtosis are relevant to the investor's decision, portfolio selection and performance measurement should be based on a four-moment framework rather than the conventional mean-variance framework. However, studies based on the four-moment framework are often limited and approaches to them vary. Studies on performance measurement mainly focus on overcoming the shortcomings of mean-variance measures by accounting for asymmetry of asset returns. Studies on portfolio selection mainly focus on maximising portfolio returns using investor functions expressed beyond the second moment. Details of these studies are presented next.

2.4.1 Performance Measures with Skewness and Kurtosis

As mentioned previously, skewness and kurtosis are important to the investor's decision and therefore mean-variance measures are not sufficient to evaluate the performance of non-normally distributed asset returns. A variety of alternative risk measures are proposed in the literature to overcome the shortcomings of mean-variance measures. For example, the Sortino ratio developed by Sortino and Price (1994) suggests that downside risk is more relevant to the investor's decision and therefore downside deviation should be used to measure the volatility of asset returns. The Prakash and Bear (PB) ratio (Prakash and Bear 1986) is constructed on the basis of a mean-variance-skewness framework. The advantages of this measure are that: it is in a form useful for empirical tests; and in the absence of skewness (i.e. returns are normally distributed), it is equivalent to traditional performance measures based on a mean-variance framework.

The Sortino ratio is defined as:

$$\text{Sortino ratio} = \frac{E(R) - r_f}{DD}, \tag{2.31}$$

where DD is the downside deviation and is measured as:

$$DD^2 = \frac{1}{N} \sum_{i=1}^{i=N} [R_t - E(R_t)]^2 , \tag{2.32}$$

where $R_t < E(R_t)$; R_t is the asset return at time t and r_f is the risk-free rate.

The Sortino ratio is thought to be a more appropriate performance measure than the Sharpe ratio where investors are risk-averse and the return distribution is skewed. This is because the Sharpe ratio measure assumes that investors are indifferent to downside losses and upside gains and therefore this measure may lead to incorrect ranking of asset performance if the asset distributions are skewed (Chaudhry and Johnson (2008)). The Sortino ratio incorporates the asymmetry of the return distribution and is written in the form of a modified Sharpe ratio where downside deviation (i.e. lower semi-variance) replaces standard deviation in the denominator.

The PB ratio is derived from the assumption that the utility follows a power function expressed as a function of wealth, W, as follows:

$$U(W) = (b - 1)^{-1}(a + W)^{1-1/b}.$$

The PB is defined as:

$$\lambda_i = \frac{E(R_i) - r_f}{\eta \mathrm{Cov}(R_i, R_m) + E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^2]}.$$

where $\eta = -\frac{2b}{b+1}(a + E(R_m))$ and a, b are parameters of the utility function.

(See Prakash and Bear (1986) for details.)

2.4.2 Portfolio Optimisation with Skewness and Kurtosis

With the presence of skewness and kurtosis, portfolio selection is a trade-off between the objectives of maximising expected return and skewness and minimising risk and kurtosis simultaneously. Arditti (1967) and Ingersoll (1975) propose that investors with decreasing absolute risk aversion have to forgo the return if they want to gain more benefit from increasing portfolio skewness and vice versa. Lai (1991) proposes a polynomial goal programming (PGP) model to achieve the portfolio's objectives in the presence of skewness and kurtosis. The important feature of PGP is that an optimal solution always exists as one of the feasible solutions. Thus, PGP helps to solve the problem of corner solutions for portfolios constructed only of bonds and stocks under downside risk criteria. The other features of PGP are its flexibility in incorporating investor preferences and its relative simplicity in terms of computational requirements. PGP is helpful in providing guidance on optimal asset allocation decisions such as which investment strategies should be included or how much capital should be allocated to each strategy. The algorithm for PGP is summarised as:

Let $X^T = [x_1, .., x_n]$ be the transposed weight vector of assets in the portfolio.

Let R be an n×1 stock return vector which its mean stock return vector is given by:

$\bar{R}^T = (\bar{R}_1, .., \bar{R}_n)$.

Let portfolio return be a function of x: $R(x) = X^T R$.

Let $V(x)$ be a variance function of portfolio returns as: $V(x) = E[X^T(R - \overline{R})]^2$.

Let $S(x)$ be a variance function of portfolio returns as: $S(x) = E[X^T(R - \overline{R})]^3$.

In the presence of skewness, portfolio optimisation is the trade-off between two set of objectives: (1) maximising expected return and positive skewness; and (2) minimising the variance. The optimal portfolio can be achieved by solving the following multi-objective problem:

Maximise $O_1 = X^T(R - r_f)$,

Minimise $O_2 = E[X^T(R - \overline{R})]^2$, and

Maximise $O_3 = E[X^T(R - \overline{R})]^3$,

subject to $X^T I = 1$ and $X \geq 0$

where O_i is the objective i and I is an n×1 unit vector.

2.5 Conclusions

This chapter has outlined the major published studies that are relevant to the area of higher moments. The general lack of studies investigating the importance of skewness and kurtosis in asset pricing has motivated this study to investigate the roles of skewness and kurtosis as pricing factors for asset returns. This study uses Australian data to address the gaps identified in the literature review in the following ways. Chapter 4 investigates the mean-variance efficiency of the CAPM and provides a theoretical framework for the inclusion of skewness and kurtosis into asset pricing. Chapter 5 investigates the validity of the four-moment model using a time-series approach. Chapter 6 examines whether systematic skewness and systematic kurtosis are important as pricing factors for asset returns using a cross-sectional approach. Finally, Chapter 7 explores the relationship between beta asymmetry and systematic skewness and systematic kurtosis and examines whether these factors can proxy for risk asymmetry caused by changes in market conditions.

CHAPTER 3. DATA AND SUMMARY STATISTICS

3.1 Data

This study uses weekly returns of 2224 stocks listed on the ASX and traded in Australian dollars from January 1992 to May 2009. As daily data contains too much noise and monthly data results in limited observations, weekly data is the favoured choice. The study uses the ASX300 index as a proxy for the Australian market. The index is the value-weighted average of the 300 largest Australian companies based on their market capitalisation. The 90-day bank-accepted bill rate is used as a proxy for the risk-free rate. To accurately reflect the performance of stocks and indices, the study uses return indices, which assume that all dividends and distributions are reinvested to compute weekly returns. The data is collected from DataStream yielding approximately 650,000 observations.

To check the robustness of the empirical tests and to investigate the impacts of the economic cycle on measures of market risk, the study considers five sub-periods: 1992–1996, 1997–2001, 2002–2006, 2007–2009 and 1992–2006.

In 1992–1996, the Australian economy recovers from the recession of 1990–1991 and grows moderately with its GPD growth peak of 4.2% per annum in 1996[1]. From 1997 the economy experiences slow growth due to the impact of the Asian financial crisis. The dot-com

[1] Source: Australian Bureau of Statistics.

bubble in 1998–2000 has only a minor impact on the market since the Australian economy relies heavily on the commodities and resources markets. However, the deflation of the dot-com bubble in 2000–2001 and the 11 September 2001 event depreciate the market to a four-year low. After 2001, the world economy moves into an expansionary phase which provides a dynamic environment for the Australian economy. Domestic demand in Australia grows at over 5 percent in 2002–2003 and 2003–2004, well above its long-term average growth[2]. The period from 2002 to 2006 sees a global explosion in prices in commodities, notably oil, food and housing. It is believed that the crash of the dot-com bubble help create the housing bubble experienced in this period as investors move their investments from the risky stock market to a safer investment market such as the housing market (Holcombe and Powell 2009). In the period of 2007–2009, the economy suffers from the global financial crises (GFC) due to the deflation of housing and commodity bubbles in many major economies. The periods of 1992–1996 and 2002–2006 are considered bull periods while the periods of 1997–2001 and 2007–2009 are considered bear periods. The study considers the period of 1992–2006 separately to examine whether the GFC had any significant impacts on the overall results of the 1992–2009 period.

3.2 Measures of Systematic Skewness and Systematic Kurtosis

This study focuses on systematic measures of skewness and kurtosis which are considered as analogs of the market beta. Kraus and Litzenberger (1976) define the systematic measure of skewness as follows:

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[2] Source: Australian Bureau of Statistics.

$$S_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^2]}{\{R_m - E(R_m)\}^3}. \qquad (3.1)$$

Using the methodology suggested by Kraus and Litzenberger (1976), this study defines systematic measure of kurtosis as:

$$K_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^3]}{\{R_m - E(R_m)\}^4}, \qquad (3.2)$$

where R_i and R_m are the return of asset i and the market return respectively.

Systematic skewness (kurtosis) is defined as a component of an asset's skewness that is related to the market portfolio's skewness (kurtosis). In this context, the systematic skewness (kurtosis) is considered as a non-diversifiable measure of skewness (kurtosis) and therefore it is consistent with the assumption of portfolio theory that only systematic risk is relevant to the investor's decision.

3.3 Portfolio Construction

This study uses portfolios constructed on the basis of systematic skewness and systematic kurtosis as underlying assets to examine the risk-return relationship. Since the goal of this study is to investigate the roles of skewness and kurtosis in asset pricing, this method of portfolio formation is important to allow the study to observe the patterns between portfolio returns and risk factors related to systematic skewness and systematic kurtosis. The portfolio formation is in the spirit of Fama and French (1992) and Harvey and Siddique (2000). The systematic skewness and systematic kurtosis of each stock are computed using the equations (3.1) and (3.2). Once the systematic skewness and the systematic kurtosis of each stock are computed, they are then ranked according to the magnitude of systematic skewness and systematic kurtosis. The stocks are sorted into five systematic skewness quintiles and five systematic kurtosis quintiles, with 445 stocks in each quintile. According to the ranking, quintile 5 contains the highest systematic skewness (kurtosis) and quintile 1 the lowest.

To construct explanatory portfolios, the study uses the method of mimicking portfolios proposed by Fama and French (1992). The explanatory portfolios in time-series and cross-sectional regressions which are used for empirical analyses in Chapters 4, 5 and 6 are returns of the market portfolio and of mimicking portfolios for the systematic skewness and systematic kurtosis factors. The study uses the ASX300 market index return minus the 90-day Bank Bill Accepted rate as a proxy for the market premium. The mimicking portfolios are formed to mimic underlying risk factors in returns related to systematic skewness and systematic kurtosis. The returns of the mimicking portfolios are the difference between the returns on the highest systematic skewness (kurtosis) portfolio (quintile 5), and the returns on the lowest systematic skewness (kurtosis) portfolio (quintile 1).

For each of the five systematic skewness quintiles, the stocks are further ranked by systematic kurtosis and sorted into five portfolios giving 25 portfolios in total. These portfolio returns are used as dependent variables for subsequent empirical tests in Chapters 4, 5 and 6. The study uses dependent portfolios formed on the basis of both systematic skewness and systematic kurtosis because the study seeks to determine whether the mimicking portfolios used as explanatory variables can capture common factors in stock returns related to systematic skewness and systematic kurtosis.

3.4 Summary Statistics

Table 3.1 presents summary statistics of 25 weekly Australian stock portfolios for the period from January 1992 to May 2009. The number of time-series observations during the period is 906 while the number of stocks in each portfolio ranges from 38 to 201. In addition to the mean and standard deviation of stock returns, the total skewness and excess kurtosis are calculated as:

$$\text{Skewness} = \frac{1}{T-1}\sum_{t=1}^{T}\left[\frac{R_{it}-\bar{R}_i}{\sigma_i}\right]^3, \text{ and} \tag{3.3}$$

$$\text{Kurtosis} = \frac{1}{T-1}\sum_{t=1}^{T}\left[\frac{R_{it}-\bar{R}_i}{\sigma_i}\right]^4 - 3\ , \tag{3.4}$$

where R_i, $\bar{R}_i$ and σ_i are the weekly returns, expected return and the standard deviation of portfolio i respectively. The excess kurtosis is obtained by subtracting the total kurtosis from 3, which is the total kurtosis of a normal distribution. A negative excess kurtosis indicates a platykurtic distribution while a positive one indicates a leptokurtic or fat-tailed distribution.

As can be seen from the table, the portfolio mean return ranges from -0.60% to 0.10 % with 20 portfolios having negative returns. The portfolio standard deviations vary significantly from 1.00 % to 8.46 %, with the majority of them less than 4%. It is observed that the standard deviations are high for the most negatively skewed and leptokurtic portfolios, i.e. portfolios 1-4 and 1-5. Of the 25 portfolios, 15 have negative skewness while all exhibit fat tails. The skewness ranges from -1.75 to 2.24 while the excess kurtosis varies significantly from 1.06 to 107.80. It is observed that portfolios with the highest positive and negative skewness are likely to have the highest excess kurtosis. Overall, the study finds that the return distributions of the 25 portfolios are leptokurtic and generally negatively skewed.

Table 3.1 Summary statistics of returns of 25 portfolios formed by systematic skewness and systematic kurtosis January 1992 – May 2009

The table reports summary statistics of 25 portfolios formed from 2234 Australian stocks listed on ASX for the period of January 1992 to May 2009. Portfolio 1-5 contains the lowest systematic skewness and highest systematic kurtosis stocks while portfolio 5-1 contains highest systematic skewness and lowest systematic kurtosis stocks. Weekly returns of each portfolio are the average weekly returns of stocks in the portfolio. Mean and standard deviation are the first two moments of the return distribution. Skewness and kurtosis are the third and fourth standardised moments of the return distribution. The number of time-series observations for each portfolio is 906 (T=906).

	Mean	**Median**	**Maximum**	**Minimum**	**Std. Dev.**	**Skewness**	**Excess Kurtosis**	**Jarque-Bera**	**Probability**	**Number of stocks**
Portfolio 1-1	-0.08%	-0.11%	4.92%	-4.50%	1.00%	-0.129	3.21	186	0.00	190
Portfolio 1-2	-0.05%	-0.05%	6.96%	-7.69%	1.53%	-0.207	2.83	310	0.00	201
Portfolio 1-3	-0.16%	-0.25%	15.53%	-14.49%	3.27%	0.088	3.17	382	0.00	59
Portfolio 1-4	-0.24%	-0.26%	109.90%	-138.66%	6.72%	-1.677	97.7	415,257	0.00	38
Portfolio 1-5	-0.60%	-0.30%	280.94%	-303.50%	8.46%	-1.749	107.80	392,728	0.00	40
Portfolio 2-1	0.02%	0.05%	3.56%	-5.61%	1.09%	-0.341	1.06	60	0.00	167
Portfolio 2-2	0.04%	0.07%	4.28%	-10.33%	1.28%	-0.694	5.34	1,151	0.00	106
Portfolio 2-3	0.01%	0.07%	11.19%	-14.65%	2.46%	-0.235	3.65	513	0.00	120
Portfolio 2-4	-0.02%	0.09%	21.47%	-13.13%	2.82%	-0.116	5.73	1,244	0.00	39
Portfolio 2-5	-0.05%	0.06%	40.35%	-27.71%	4.98%	0.495	8.33	2,656	0.00	42
Portfolio 3-1	-0.03%	-0.06%	63.58%	-30.28%	5.12%	2.241	8.80	43,760	0.00	68
Portfolio 3-2	0.10%	0.16%	8.69%	-10.16%	1.64%	-0.161	4.36	723	0.00	120
Portfolio 3-3	0.00%	0.15%	8.90%	-12.79%	1.95%	-1.069	5.35	1,254	0.00	105
Portfolio 3-4	-0.05%	0.14%	16.55%	-12.72%	2.43%	-0.700	5.64	1,274	0.00	69
Portfolio 3-5	-0.10%	0.02%	34.13%	-22.84%	4.40%	0.197	6.48	1,593	0.00	55
Portfolio 4-1	-0.40%	-0.56%	52.81%	-38.02%	6.54%	0.680	9.81	3,498	0.00	42
Portfolio 4-2	-0.02%	0.05%	18.55%	-15.34%	3.47%	-0.106	2.67	269	0.00	44
Portfolio 4-3	-0.04%	0.11%	19.62%	-18.32%	3.01%	0.081	5.80	1,749	0.00	151
Portfolio 4-4	-0.07%	0.11%	12.03%	-14.99%	2.67%	-0.800	4.41	831	0.00	57
Portfolio 4-5	-0.08%	0.07%	25.28%	-20.26%	3.27%	-0.572	9.79	3,673	0.00	154
Portfolio 5-1	-0.30%	-0.20%	30.95%	-32.63%	5.70%	-0.253	3.62	493	0.00	44
Portfolio 5-2	-0.36%	-0.23%	33.65%	-33.65%	6.52%	0.095	4.95	852	0.00	52
Portfolio 5-3	-0.18%	-0.19%	32.84%	-18.49%	3.89%	0.419	7.34	2,061	0.00	46
Portfolio 5-4	-0.23%	0.00%	10.45%	-22.82%	3.24%	1.326	10.31	2,287	0.00	103
Portfolio 5-5	-0.18%	0.17%	15.15%	-28.61%	3.59%	1.881	12.10	6,061	0.00	112

To investigate the normality assumption, the Jarque-Bera normality test is applied. The Jarque-Bera (JB) statistic compares skewness and kurtosis of the return series with those of the normal distribution[3]. The rejection of the normality hypothesis in every portfolio reinforces the suggestions of studies such as Harvey and Zhou (1993) and Richardson and Smith (1993) which propose that portfolio returns do not conform to a normal distribution. Overall, this result provides strong preliminary support for the mean-variance efficiency tests in the following chapter.

Table 3.2 presents systematic estimates of skewness and kurtosis for the 25 portfolios. The study defines systematic skewness and systematic kurtosis as standardised estimates of skewness and kurtosis risk analogous to the CAPM beta. The systematic skewness and systematic kurtosis of the portfolios are measured using equations (3.1) and (3.2) in page 44. Panel A reports the systematic skewness of the 25 portfolios. The systematic skewness varies from -2.65 to 2.17 with 12 portfolios having negative systematic skewness. The portfolios in the lowest systematic skewness quintile have the most negative systematic skewness while the portfolios in the highest systematic skewness quintile have the most positive systematic skewness. If systematic skewness risk is found to contribute significantly to the return premium, the study expects to find that these negatively skewed portfolios are compensated with a higher systematic skewness premium compared to those with positive systematic skewness. This is because investors demand a higher premium for holding stocks with large downside risk while

[3] $JB = \frac{n}{6}\left(S^2 + \frac{1}{4}K^2\right)$ where n is the number of observations, S is the total skewness and K is the excess kurtosis of portfolio i. JB has an asymptotic Chi-square distribution with two degrees of freedom.

they are willing to pay a premium for stocks in upside gains. Finally, the study observes that controlling for the systematic skewness effect, the systematic skewness of the portfolios tends to increase from the lowest to the highest kurtosis portfolios; and controlling for the systematic kurtosis effect, the systematic skewness generally increases from the lowest to the highest skewness portfolios.

In panel B, the systematic kurtosis varies from -0.40 to 1.40 with five portfolios in the highest kurtosis quintile also having the highest systematic measures of kurtosis. Consistent with the patterns observed in panel A, the systematic kurtosis increases monotonically from the lowest to the highest systematic kurtosis portfolios once the systematic skewness effect is controlled. On the other hand, controlling for the systematic kurtosis effect, the systematic kurtosis does not vary consistently from the lowest to the highest systematic skewness portfolios. Finally, the study notes that the portfolios with the most negative systematic skewness are also those with the highest systematic kurtosis, i.e. portfolios 1-4 and 1-5.

Table 3.2 Systematic skewness and systematic kurtosis estimates of 25 portfolios formed on the basis of systematic skewness and systematic kurtosis

The table reports the estimated systematic skewness and systematic kurtosis of the 25 portfolios formed on the basis of systematic skewness and systematic kurtosis quintiles. The systematic skewness and systematic kurtosis are computed as follows:

$S_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^2]}{\{R_m - E(R_m)\}^3}$; $K_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^3]}{\{R_m - E(R_m)\}^4}$ where R_i and R_m are the return of asset i and the market return respectively.

Panel A: Systematic Skewness

	Low Sys. Kurtosis	2	3	4	High Sys. Kurtosis
Low Sys. Skewness	-1.041	-0.937	-1.077	-2.378	-2.646
2	-0.514	-0.515	-0.437	-0.486	-0.267
3	-0.118	-0.008	0.109	0.052	0.293
4	0.850	0.843	0.906	0.704	0.719
High Sys.Skewness	1.996	1.988	1.670	1.799	2.169

Panel B: Systematic Kurtosis

	Low Sys. Kurtosis	2	3	4	High Sys. Kurtosis
Low Sys. Skewness	-0.306	-0.064	0.261	1.286	1.398
2	-0.247	-0.045	0.256	0.540	1.015
3	-0.286	0.004	0.248	0.525	1.116
4	-0.228	-0.015	0.333	0.599	1.068
High Sys.Skewness	-0.401	-0.020	0.224	0.614	1.209

3.5 Summary

This chapter describes the data set which will be used for the empirical analyses in the rest of the study. It shows that returns of Australian stocks do not conform to a normal distribution and that systematic skewness and systematic kurtosis are related to other statistics. For example, the most negatively skewed and leptokurtic portfolio returns exhibit high standard deviations. Controlling for the kurtosis effect, the systematic skewness monotonically increases from the lowest to the highest skewness portfolios while the systematic kurtosis monotonically increases from the lowest to the highest kurtosis portfolios once the skewness effect is controlled. The evidence that Australian returns are leptokurtic and generally negatively skewed provides the motivation for the study to find whether mean and variance are sufficient to describe asset returns and to determine whether higher moments of the return distribution such as skewness and kurtosis are important in asset pricing.

CHAPTER 4. ARE ASSET RETURNS MEAN-VARIANCE EFFICIENT?

4.1 Introduction

The overall aim of this study is to investigate the roles of systematic skewness and systematic kurtosis in explaining the variation of expected returns. To establish a foundation for the emergence of skewness and kurtosis into asset pricing, it is important to test whether asset returns are mean-variance efficient. If mean and variance are not sufficient to describe asset returns, given that skewness and excess kurtosis are associated with non-normalities in asset returns, they may become important in asset-pricing models for explaining the return variation. Therefore, the goal of this chapter is to answer the first research question of whether mean and variance are statistically efficient in describing patterns of asset returns, that is, to investigate if the CAPM is able to explain patterns of asset returns efficiently.

Since the work of Gibbons (1982), empirical tests of the CAPM are usually conducted within the multivariate linear regression (MLR) framework. Empirical studies such as Shanken (1982, 1985 and 1992) and Shanken and Zhou (2007) suggest that the multivariate approach can lead to more appropriate conclusions than those based on traditional inference which relies on a set of dependent univariate statistics. Statistical inference for the MLR model in econometrics and empirical finance is traditionally based either on asymptotic approximations or on finite-sample distribution theory. This study uses both approaches, the Wald test, an asymptotic method, and the Gibbons, Ross and Shanken (GRS) (1989), a finite sample method, to test the mean-variance efficiency of asset returns. The study also uses the bootstrap method to test the robustness of inferences drawn from the Wald and the GRS tests.

Using Australian data from 1992 to 2009, the results from the Wald and GRS tests firmly reject the hypothesis of mean-variance efficiency of asset returns. This is clear evidence that the market beta alone is not sufficient to explain the expected returns. Although the results are consistent with conclusions drawn for the U.S. market in studies of Black, Jensen and Scholes (1972), MacKinlay and Richardson (1991) and Chou and Zhou (2006), they contradict earlier Australian results reported by Faff (1991) and Wood (1991).

As the Wald test and the GRS test for mean-variance efficiency are based on the assumption that the errors of the CAPM are multivariate normal, it is important to check if inferences drawn from these tests are robust when this assumption is violated. The study proposes a bootstrap test as a robustness test for conclusions drawn from the Wald and GRS tests. This method relies on the empirical distribution of the errors and therefore relaxes the normality assumption. Bootstrapping uses the same theory underlying Monte Carlo simulation methods, except that it uses resamples from the original data rather than from the assumed population. When the sample size is large, the bootstrapped estimates converge to the true parameters as the number of repetitions increases.

Using the bootstrap method, the study finds that the conclusions drawn from the GRS test are consistent in both cases of the error distributions. On the other hand, the conclusions drawn from the Wald test are not consistent in these cases. The finding from the Wald test is in fact consistent with the argument of Gibbons, Ross and Shanken (1989) that the Wald test may provide seriously misleading results if the error terms depart from normalities. As the GRS-statistic follows an F-distribution, the finding from the GRS test in this study is consistent with the conclusion drawn from MacKinlay (1985), which shows that the F-test is fairly robust when the sample size is large even if the error distribution is not normal.

Overall, the results generated from the mean-efficiency tests strongly indicate that asset returns are not mean-variance efficient and the market beta alone is not sufficient to explain the variation in expected returns. As asset returns are often observed to be skewed and leptokurtic, in the next chapters the study investigates whether asset returns are efficient in the mean-variance-skewness-kurtosis framework.

The rest of the chapter is organized as follows. Section 2 of this chapter illustrates methodology for mean-variance efficient tests. Section 3 presents empirical results and discussion. Section 4 is the conclusion of the chapter.

4.2 Methodology

In this section, the study first presents two standard multivariate linear regression approaches to mean-variance efficiency tests. As these approaches rely on the assumption of multivariate normal errors, the study also presents a bootstrap test as a robustness test of the conclusions drawn from the mean-variance tests. Details of these tests are as follows.

4.2.1 Mean-Variance Efficiency Tests

The Sharpe-Lintner CAPM has three testable implications:

(1) the intercept is zero;

(2) beta completely captures the cross-sectional variation of expected returns; and

(3) the market risk premium is positive.

In much of this chapter, this study carries out empirical tests of the first implication. The last two implications will be considered in later chapters.

Consider the standard multivariate CAPM:

$$r_t = \alpha + \beta r_{mt} + \varepsilon_t; \quad \varepsilon_t \sim P(0, \Sigma) \text{ and } t = 1, .., T, \tag{4.1}$$

where $\alpha = (\alpha_1, .., \alpha_N)'$; $\beta = (\beta_1, .., \beta_N)'$; $r_t = (r_{1t}, .., r_{Nt})'$ and $\varepsilon_t = (\varepsilon_{1t}, .., \varepsilon_{Nt})'$; where N is the number of underlying assets. The error term ε_t follows an unknown distribution function $P(0, \Sigma)$, whose mean is zero and variance-covariance matrix is Σ.

The mean-variance efficiency of the CAPM implies that $E(r_t) = \beta E(r_{mt})$. This imposes a restriction on the intercept α, that is $\alpha = 0$ for all i=1,..,N. From this restriction, the efficiency test of the CAPM is equivalent to testing the joint hypothesis of zero intercepts as follows:

Ho: $\alpha = 0$

against the alternative hypothesis:

H_1: $\alpha \neq 0$.

This study develops tests of joint zero intercepts under two alternative distributions of the error term: P is multivariate normal and P is multivariate non-normal.

4.2.2 Mean-Variance Efficiency with Multivariate Normal Errors

In this section, the study considers the case when the error terms are assumed to be multivariate normal with mean zero and a constant covariance matrix over time. Two traditional efficiency tests under this condition are considered as follows.

4.2.2.1 The Wald Test

The W-statistic for joint hypothesis of zero intercepts for the CAPM is specified as:

$$W = \hat{\alpha}'[\mathrm{Var}(\hat{\alpha})]^{-1}\hat{\alpha}$$

$$= h^{-1}\,\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\,, \qquad (4.3)$$

where $h = \frac{1}{T}\left(1 + \hat{\theta}^2\right)$ given $\hat{\theta} = {}^{\bar{r}_m}/_{s}$;

$\bar{r}_m$ is the sample mean of r_{mt} and s is the sample standard deviation of r_{mt};

$\hat{\alpha} = (\hat{\alpha}_1, .., \hat{\alpha}_N)'$, and

$\hat{\Sigma}$ is the sample variance-covariance matrix of ε_t where $\varepsilon_t = (\varepsilon_{1t}, .., \varepsilon_{Nt})'$.

Under the null hypothesis, W will have a chi-square distribution with N degrees of freedom.

4.2.2.2 The Gibbon, Ross, Shanken (1989) Test of Zero Intercept

An alternative to the Wald test is the GRS test developed by Gibbon, Ross, Shanken (1989), who demonstrate that this adjusted version of Wald statistic has an exact F distribution. To test the joint hypothesis of zero intercepts for two-moment CAPM, the GRS-statistic is specified as:

$$\mathrm{GRS-statistic} = \left(\frac{T}{N}\right)\left(\frac{T-N-1}{T-2}\right)\left(1+\bar{r}_m{}'\hat{\Omega}^{-1}\bar{r}_m\right)^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\ , \qquad (4.4)$$

where $\bar{r}_m$ is the sample mean of r_{mt};

$\hat{\Omega}$ is the sample variance of r_{mt};

$\hat{\alpha} = (\hat{\alpha}_1, .., \hat{\alpha}_N)'$ and

$\hat{\Sigma}$ is the sample variance-covariance matrix of ε_t where $\varepsilon_t = (\varepsilon_{1t}, .., \varepsilon_{Nt})'$.

Under the null hypothesis, the GRS statistic will have an F-distribution with degrees of freedom N and (T-N-1).

4.2.3 Bootstrap Tests with Multivariate Non-normal Errors

The assumption that the error term is multivariate normal is critical for the Wald and GRS tests. However, these assumptions do not usually hold, and statistics generated from these tests may fail to give satisfactory results. To overcome the problem of non-normality of the errors, the study uses the bootstrap method which is based on sampling from the actual residuals. Using the general methodology for bootstrapping proposed by Cho and Zhou (2006), the study designs the bootstrap efficiency test as follows:

1. Estimate the multivariate CAPM using the OLS method to obtain $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\varepsilon}_t$.

$$r_t = \hat{\alpha} + \hat{\beta} r_{mt} + \hat{\varepsilon_t} \qquad (4.5)$$

2. Calculate the Wald-statistic and GRS-statistic as follows:

$$\mathrm{W-statistic} = h^{-1}\ \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \qquad (4.6)$$

$$\mathrm{GRS-statistic} = \left(\frac{T}{N}\right)\left(\frac{T-N-1}{T-2}\right)\left(1+\bar{r}_m{}'\hat{\Omega}^{-1}\bar{r}_m\right)^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \qquad (4.7)$$

3. Repeat the following steps a large number of times (5000 for this study):

a. Resample $\hat{\varepsilon}_t^*$(t=1,..,N) from $\{\hat{\varepsilon}_t\}_{t=1}^T$ achieved from step 1 with replacement.

b. Generate simulated excess returns under the null hypothesis (intercept=0):

$$r_t^* = \beta r_{mt} + \hat{\varepsilon}_t^* \quad t = 1, .., T \tag{4.8}$$

Where $\beta = \hat{\beta}$; $\hat{\varepsilon}_t^*$ is generated from step a and $\hat{\beta}$ is generated from step 1.

c. Regress r_t^* on r_{mt} using the OLS method to obtain the estimates $\hat{\alpha}^*$and $\hat{\Sigma}^*$. Re-calculate Wald-statistic and GRS-statistic as follows:

$$W - \text{statistic}^* = h^{-1}\, \hat{\alpha}^{*\prime}\, \hat{\Sigma}^{*-1} \hat{\alpha}^* \tag{4.9}$$

$$GRS - \text{statistic}^* = \left(\frac{T}{N}\right)\left(\frac{T-N-1}{T-2}\right)\left(1 + \bar{r}_m{}'\hat{\Omega}^{-1}\bar{r}_m\right)^{-1} \hat{\alpha}^{*\prime}\, \hat{\Sigma}^{*-1} \hat{\alpha}^* \tag{4.10}$$

4. Calculate the percentage of $W - \text{statistics}^*$ ($GRS - \text{statistics}^*$) that are greater than the $W - \text{statistic}$ ($GRS - \text{statistic}$) obtained in step 2. The percentage is the p-value of the bootstrap test.

4.3 Empirical Results and Discussion

In this section, the study first presents OLS regression results generated from the CAPM. Using these results, mean-variance efficiency tests are examined. Results and discussion from the multivariate tests are then presented. Finally, results and discussion for robustness tests using the bootstrap method finish the section.

4.3.1 Time-series Regression Analysis using the CAPM

Before examining the mean-variance efficiency of the CAPM, the study first conducts a time-series regression analysis to examine the power of the market factor in explaining the variation of time-series returns. The average excess returns of the 25 portfolios that serve as dependent variables give perspectives on the range of average returns that the market factor must explain. The regression results are presented in table 4.1.

The market beta of the CAPM is positive and statistically significant at the 1 percent level of significance in every portfolio. The betas range from 0.49 to 1.47 with six portfolios having betas greater than 1. As the market beta is viewed as the sensitivity indicator of asset returns to changes in market returns, if market returns change by 1%, asset returns would follow by changes from 0.49% to 1.47%. This indicates that the market beta is not only statistically significant but also economically significant. The result supports the well-known findings of Markowitz (1952), Sharpe (1964) and Lintner (1965) that the market factor is important in explaining the variation of asset returns. However, this evidence does not imply that the CAPM holds, as the CAPM predicts that no variables other than the market beta should explain a firm's expected returns. In the next section, the study examines if the market factor is the only important variable in explaining the variation of asset returns.

Table 4.1 CAPM regression results of 25 portfolios formed by systematic skewness and systematic kurtosis for the period of 1992-2009

The table presents regressions results for the CAPM: $r_t = \alpha + \beta r_{mt} + \varepsilon_t$ where r_t is the asset excess return at time t, r_{mt} is the excess return of the market index at time t. Student's t-statistics are reported in the parentheses below the coefficient estimates. * and ** denote statistical significance at 5 and 1 percent levels.

	Low S. Kurtosis	2	3	4	High S. Kurtosis
			Intercept (α)		
Low S. Skewness	-0.0013 (-2.41)*	-0.0014 (-2.27)*	-0.0029 (-2.80)**	-0.0028 (-2.03)*	-0.0075 (-2.99)**
2	-0.0003 (-0.49)	-0.0005 (-0.86)	-0.0010 (-1.27)	-0.0024 (-1.98)*	-0.0020 (-2.05)*
3	-0.0007 (-0.43)	-0.0001 (-0.11)	-0.0022 (-1.96)*	-0.0016 (-2.20)*	-0.0023 (2.25)*
4	-0.0045 (-2.17)*	-0.0009 (-0.78)	-0.0023 (-1.99)*	-0.0020 (-2.57)**	-0.0022 (-2.41)**
High S. Skewness	-0.0038 (-1.97)*	-0.0043 (-2.08)*	-0.0025 (-2.04)*	-0.0039 (-3.89)**	-0.0034 (-3.33)*
			Market Premium (β)		
Low S. Skewness	0.5584 (17.13)**	0.7519 (21.83)**	0.9213 (18.22)**	1.2828 (6.13)**	1.4697 (4.33)**
2	0.5381 (15.09)**	0.6778 (22.25)**	0.8566 (22.65)**	1.1817 (21.21)**	1.2300 (11.83)**
3	0.4872 (6.49)**	0.6358 (18.57)**	0.8196 (24.46)**	1.0509 (22.39)**	1.0919 (14.52)**
4	0.5384 (5.75)**	0.6114 (11.21)**	0.7306 (14.46)**	0.9120 (24.14)**	0.9888 (19.91)**
High S. Skewness	0.5159 (5.27)**	0.6092 (6.45)**	0.7448 (11.11)**	0.9217 (16.98)**	0.9336 (18.87)**
			Adjusted R-squared		
Low S. Skewness	0.486	0.530	0.374	0.172	0.125
2	0.451	0.598	0.470	0.467	0.249
3	0.199	0.512	0.566	0.515	0.313
4	0.150	0.191	0.357	0.509	0.474
High S. Skewness	0.129	0.153	0.202	0.398	0.442

It is observed that the market beta is lowest for the highest skewness quintile (quintile 5), and highest for the lowest skewness quintile (quintile 1). Conversely, the beta is lowest for the lowest kurtosis quintile and highest for the highest kurtosis quintile. As documented in Chapter 3, returns of portfolios in the lowest skewness quintile are the most negatively skewed while returns of portfolios in the highest kurtosis quintile are the most leptokurtic. This evidence implies that negatively skewed and leptokurtic asset returns are likely to have a high market beta. Moreover, these negative beta-skewness and positive beta-kurtosis relationships are consistent with suggestions from the earliest studies of higher moments. For example, the studies of Arditti (1967), Ingersoll (1975) and Arrow (1971) suggest that risk-averse investors have positive preferences towards odd moments (e.g. return and skewness) and negative preferences towards even moments (e.g. variance and kurtosis). Consequently, risk-averse investors who prefer low market beta assets would also prefer low kurtosis and high positive skewness assets and vice versa. Finally, table 4.1 shows that the intercepts of the CAPM are significantly different from zero in 19 of the 25 portfolios examined. The result provides some preliminary support for the multivariate test of joint zero CAPM intercepts to examine the mean-variance efficiency, which is presented in the following section.

4.3.2 Mean-Variance Efficiency Tests with Normal Errors

To examine the mean-variance efficiency of the CAPM, this section focuses on testing the hypothesis of multivariate zero intercepts using the standard Wald test and the GRS test. These tests rely on the assumptions that the error terms are multivariate normal with a zero mean and a constant variance-covariance matrix. To compute the Wald-statistic and the GRS-statistic, estimates of the CAPM parameters are first obtained by using the OLS method. Using residuals from the OLS regressions, the study constructs an estimate of the variance-covariance matrix of the error terms. The Wald-statistic and the GRS-statistic are computed using equation (4.3) in page 53 and equation (4.4) in page 54. The Wald-statistic follows a Chi-square distribution with N degrees of freedom while the GRS-statistic has a non-central F distribution with degrees of freedom N and T-N-1. T is the number of time-series observations and N is the number of the underlying assets.

Table 4.2 Mean-Variance Efficiency Tests with Normal Errors

The table reports Wald-statistics and GRS-statistics of multivariate tests of zero intercepts of the CAPM. The multivariate CAPM is specified as $r_t = \alpha + \beta r_{mt} + \varepsilon_t$ where $\alpha = (\alpha_1, .., \alpha_N)'; \beta = (\beta_1, .., \beta_N)'; r_t = (r_{1t}, .., r_{Nt})'$ and $\varepsilon_t = (\varepsilon_{1t}, .., \varepsilon_{Nt})'$. N is the number of underlying assets and ε_t is assumed to be multivariate normal. The Wald and GRS-statistics are calculated as follows:

$\mathrm{W-stastistic} = h^{-1}\,\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$ where $h = \frac{1}{T}\left(1+\hat{\theta}^2\right)$; $\hat{\theta} = \bar{r}_m / s_m$; $\hat{\alpha} = (\hat{\alpha}_1, .., \hat{\alpha}_N)'$ and $\hat{\Sigma}$ is the sample variance-covariance matrix of ε_t. W-statistic follows a Chi-square distribution with N degrees of freedom.

$\mathrm{GRS-statistic} = \left(\frac{T}{N}\right)\left(\frac{T-N-1}{T-2}\right)\left(1+\bar{r}_m'\hat{\Omega}^{-1}\bar{r}_m\right)^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$ where $\bar{r}_m$ is the sample mean of r_{mt}; $\hat{\Omega}$ is the sample variance of r_{mt}. The GRS-statistic follows an F-distribution with degrees of freedom N and (T-N-1).

* and ** denote statistical significance at 5 and 1 percent levels.

Period	W-statistic	GRS-statistic
1992-1996	38.95	1.512
(P-value)	(0.037)*	(0.061)
1997-2001	39.26	1.650
	(0.033)*	(0.031)*
2002-2006	44.81	1.703
	(0.009)**	(0.023)*
2007-2009	75.14	2.454
	(0.000)**	(0.001)**
1992-2006	40.56	1.727
(Period before the GFC)	(0.012)*	(0.015)*
1992-2009	48.69	1.896
	(0.003)**	(0.005)**

Table 4.2 presents the results of the mean-variance efficiency tests using the Wald and GRS tests. To check the robustness of the results, these tests are also performed for five sub-periods: 1992–1996, 1997–2001, 2002–2006, 2007–2009 and 1992–2006. When the error term is normal, the Wald test and the GRS test are statistically equivalent as they provide similar p-values except for the 1992–1996 period. The Wald test rejects the null hypothesis that the multivariate CAPM intercepts are jointly zero at the 5 percent level of significance in the entire period of 1992–2009 and in all sub-periods. The GRS test indicates that the null hypothesis is rejected at the 5 percent level for the entire period of 1992–2009 and four of the five sub-periods but not for the period of 1992–1996. Nevertheless, at the 10 percent level of significance, the hypothesis is rejected for the 1992–1996 period. This is clear evidence that the market beta alone is not sufficient to explain the expected returns. The results are consistent with those reported in the U.S. market by Black, Jensen and Scholes (1972), MacKinlay and Richardson (1991) and Chou and Zhou (2006). However, the results contradict some earlier Australian results reported by Faff (1991) and Wood (1991). In the next section, the study further investigates if these results are still persistent when the assumption of normal errors is violated.

4.3.3 Bootstrap Test with Non-normal Errors

In this section, the study evaluates the effects of non-normal errors in the mean-variance efficiency tests presented in the previous section. This is a robustness check of conclusions drawn from the Wald test and the GRS test as the error terms of the CAPM are non-normal. Bootstrap techniques are used to determine the effects of these errors on the mean-variance efficiency tests. The study first estimates the multivariate CAPM using the OLS method and then draws a sample with replacement from the residuals of the OLS regressions. Using this new set of residuals, the study generates a new set of simulated excess returns of the dependent variable under the null hypothesis of multivariate zero intercepts. OLS regressions are re-run based on the new simulated excess returns and the market premium. Using estimates of these regressions, the study recalculates the Wald-statistic and the GRS-statistic. This procedure is repeated 5000 times to build up empirical distributions of these statistics, which can be used to compare with the Wald and GRS-statistics derived from the efficiency tests with normal errors. The p-value generated from the bootstrap method is the percentage of new Wald or GRS-statistics that are greater than the original Wald or GRS-statistics.

Table 4.3 provides summary statistics of the Wald-statistics and the GRS-statistics generated from the bootstrap method. In general, the mean and median of the GRS-statistic and the Wald-statistic do not vary substantially across the periods, except for the 2007–2009 period. The standard deviation of the GRS-statistic is, on average, only about a quarter of the mean while the standard deviation of the Wald-statistic is, on average, about a third of the mean. However, as the GRS-statistic follows a non-central F-distribution and the Wald-statistic follows a Chi-square distribution, the mean-standard deviation relationship in these contexts does not provide much inference for the risk-return relationship. Figures 4.1 and 4.2 reveal the distributions of the bootstrapped Wald-statistics and GRS-statistics.

Table 4.3 Summary statistics of bootstrapped Wald-statistic and GRS-statistic distributions for the period of January 1992- May 2009

The table reports the summary statistics of the bootstrapped Wald-statistic and GRS-statistic for the period of January 1992 to May 2009 and five sub-periods: 1992–1996, 1997–2001, 2002–2006, 2007–2009 and 1992–2006. The GRS-statistic follows an F-distribution with the degrees of freedom of N and T-N-1. The Wald-statistic follows a Chi-square distribution with the degrees of freedom of N. T is the number of observations and N is the number of portfolios. The number of repetitions is 5000 in every case.

	1992-1996	1997-2001	2002-2006	2007-2009	1992-2006	1992-2009
			GRS-statistic			
Mean	1.041	1.049	1.055	1.244	1.082	1.091
Median	1.007	1.006	1.015	1.175	1.057	1.055
Maximum	2.430	3.095	2.464	4.743	2.658	2.711
Minimum	0.042	0.220	0.263	0.323	0.305	0.281
Std. Dev.	0.299	0.326	0.325	0.469	0.310	0.312
Skewness	0.617	0.793	0.674	1.213	0.538	0.598
Kurtosis	3.673	4.187	3.553	6.041	3.317	3.495
			Wald-statistic			
Mean	33.91	28.90	29.08	38.73	27.90	28.02
Median	32.40	27.73	27.97	36.59	27.26	27.09
Maximum	87.06	85.28	67.89	147.64	68.58	69.63
Minimum	7.21	6.06	7.26	10.05	7.86	7.23
Std. Dev.	11.10	9.00	8.98	14.60	8.00	8.03
Skewness	0.888	0.793	0.674	1.213	0.538	0.598
Kurtosis	4.290	4.187	3.554	6.041	3.317	3.495

Figure 4.1 Bootstrap Distributions of GRS-statistics

The figure presents bootstrap distributions of GRS-statistics for the period of 1992–2009 and five sub-periods: 1992–1996, 1997–2001, 2002–2006, 2007–2009 and 1992–2006.The GRS-statistic statistic sample is generated from the bootstrap method with the number of repetitions of 5000.

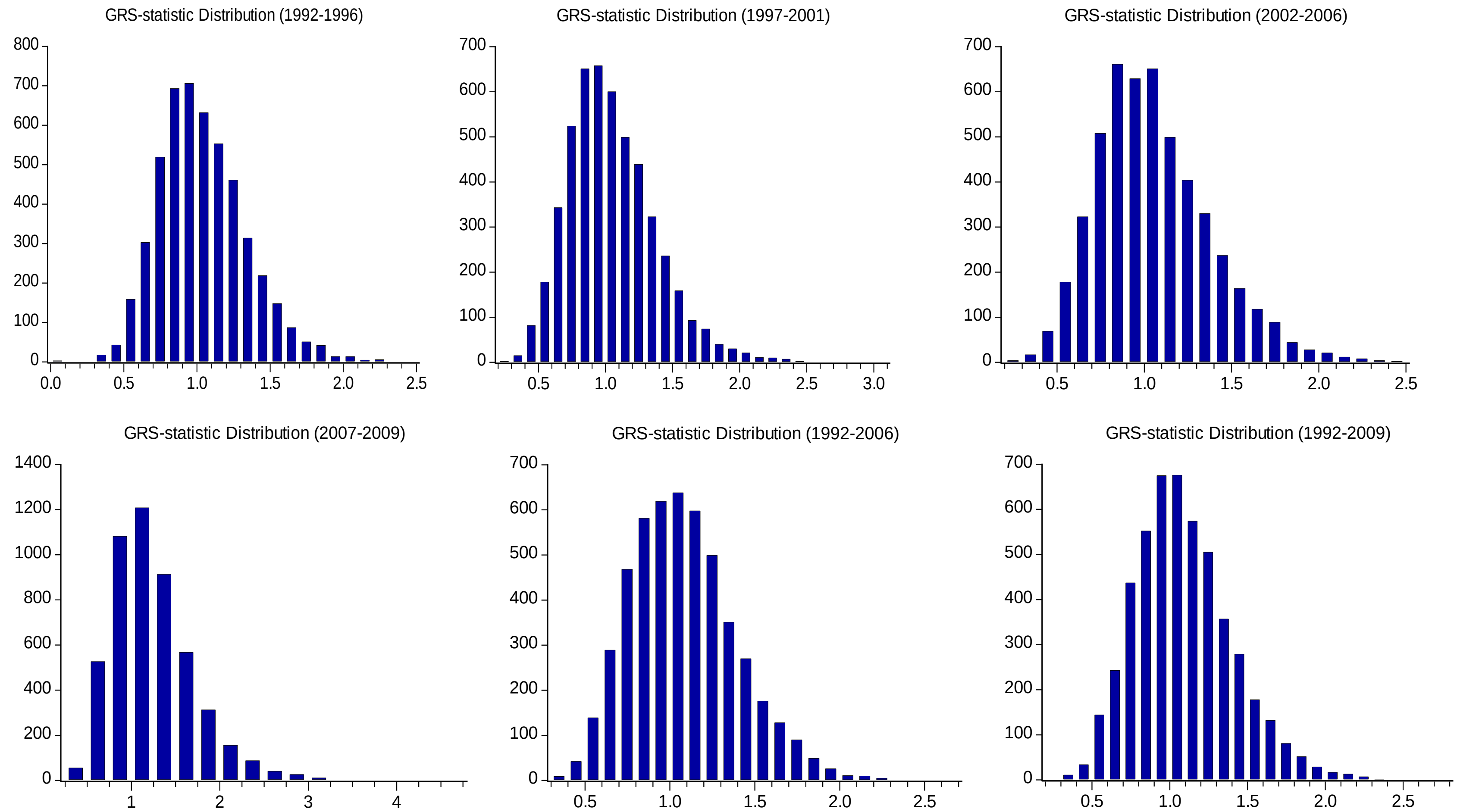


Figure 4.2 Bootstrap Distributions of Wald-statistics

The figure presents bootstrap distributions of Wald-statistics for the period of 1992-2009 and 5 sub-periods: 1992-1996, 1997-2001, 2002-2006, 2007-2009 and 1992-2006. The Wald-statistic sample is generated from the bootstrap method with the number of repetitions of 5000.

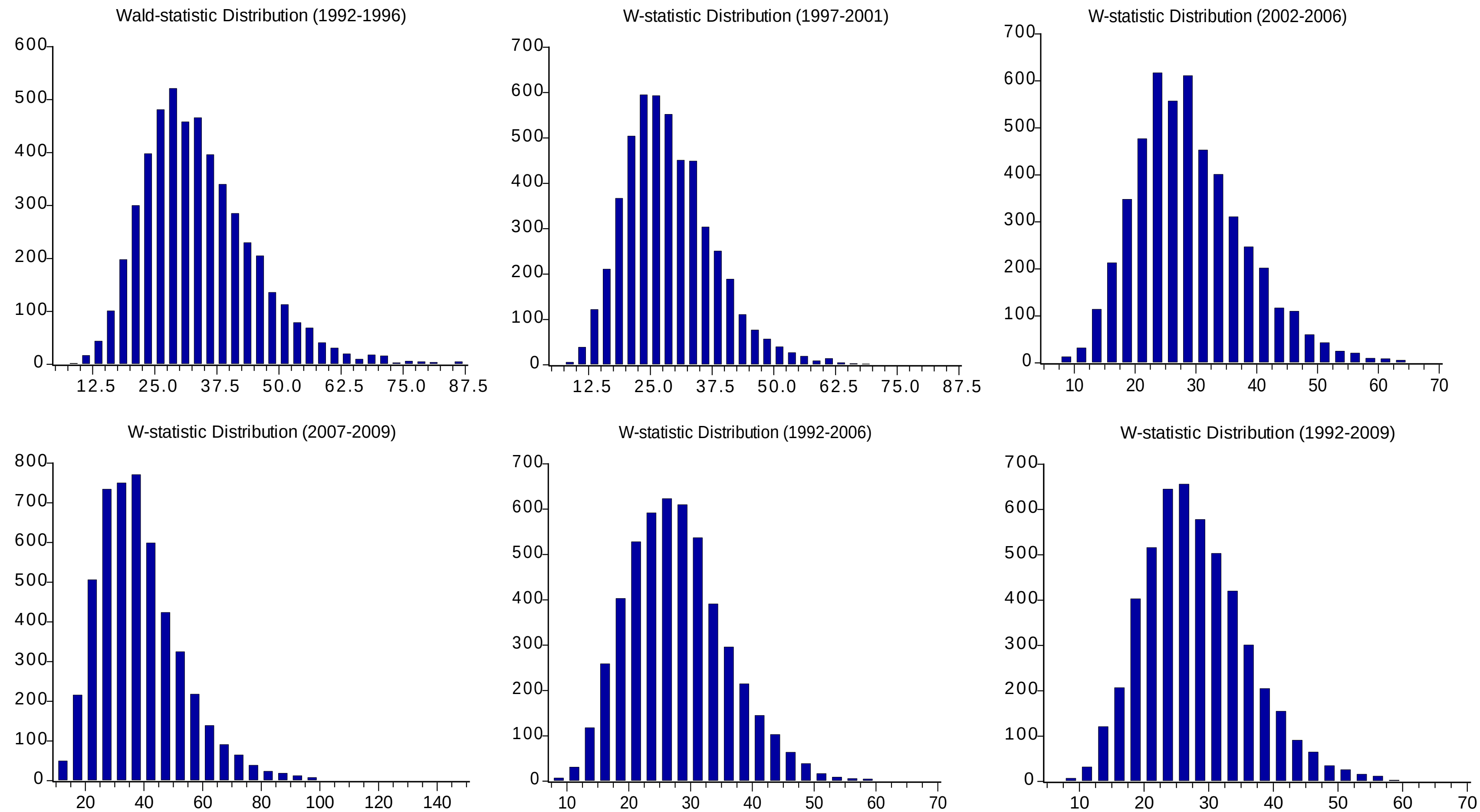


Table 4.4 presents p-values of the bootstrap tests and compares these values with the p-values of the Wald and GRS tests when the errors are multivariate normal. Overall, the null hypothesis of joint zero intercepts is rejected in most of the periods using the bootstrapped GRS test but it is not rejected using the bootstrapped Wald test. The results for the GRS test are consistent between two cases of error distributions. On the other hand, the bootstrapped Wald-statistic suggests that the Wald test is significantly influenced by the non-normality properties of the errors. The table also shows that the bootstrapped results of the Wald test are not consistent with its results generated from the assumption of normal errors. Using the Wald test for two sub-periods, 1992-1996 and 1997-2001, the hypothesis of joint zero intercepts is no longer rejected and it is marginal in the third period of 2002-2006. The finding is supported by the argument proposed by Gibbons, Ross and Shanken (1989) that Wald-statistic inferences can be seriously misleading if the error terms depart from normality. The robustness of the GRS tests in both scenarios of the errors is consistent with the studies of MacKinlay (1985) and of Gibbons, Ross and Shanken (1989) who use simulation evidence to suggest that the F-test is fairly robust when the sample size is large and when the errors deviate from normality. Overall, there is sufficient evidence to conclude that asset returns are not mean-variance efficient and therefore the CAPM is not efficient.

Table 4.4 Bootstrapping Mean-Variance Efficiency Tests with Multivariate Non-normal Errors

The table reports p-values of the multivariate tests of zero intercepts of the CAPM using the bootstrap method. The multivariate CAPM is specified as $r_t = \alpha + \beta_i r_{mt} + \varepsilon_t$ where $\alpha = (\alpha_1, .., \alpha_N)$; $\beta = (\beta_1, .., \beta_N)$; $r_t = (r_{1t}, .., r_{Nt})$; $\varepsilon_t = (\varepsilon_{1t}, .., \varepsilon_{Nt})$ and ε_t is assumed to be non-normal. First, the multivariate CAPM estimates are generated from the OLS method. Based on the OLS estimates, the Wald-statistic and GRS-statistic are calculated as: $W - \text{statistic} = h^{-1}\, \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}$ where $h = \frac{1}{T}\left(1 + \hat{\theta}^2\right)$; $\hat{\theta} = \bar{r}_m / s_m$; $\bar{r}_m$ is the sample mean and s_m is the sample standard deviation of r_{mt}; $\text{GRS} - \text{statistic} = \left(\frac{T}{N}\right)\left(\frac{T-N-1}{T-2}\right)\left(1 + \bar{r}_m' \hat{\Omega}^{-1} \bar{r}_m\right)^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}$ where $\hat{\Omega}$ is the sample variance of r_{mt} and $\hat{\Sigma}$ is the variance-covariance matrix of the error terms. A new error sample with replacement is generated from the OLS residuals. Based on the new error sample, a new excess return series is generated under the null hypothesis. Regressing the new excess returns on market returns generates a new set of multivariate CAPM estimates. Based on these new estimates, Wald and GRS-statistics are recalculated. The number of repetitions is 5000 in every case. The p-value of the bootstrap method is the percentage of new Wald or GRS statistics that are greater than the original Wald or GRS statistics. * and ** denote statistical significance at 5 and 1 percent levels.

	Normal errors		**Non-normal errors**	
Period	**Wald test P-value**	**GRS test P-value**	**Bootstrapped Wald test P-value**	**Bootstrapped GRS test P-value**
1992-1996	0.037*	0.061	0.281	0.069
1997-2001	0.033*	0.031*	0.135	0.048*
2002-2006	0.009**	0.023*	0.051	0.037*
2007-2009	0.000**	0.001**	0.022*	0.018*
1992-2006 (Period before the GFC)	0.012*	0.015*	0.025*	0.032*
1992-2009	0.003**	0.005**	0.014*	0.013*

4.4 Conclusions

In this chapter, the study has successfully shown that asset returns are not mean-variance efficient for the overall study period of 1992–2009 and most of the sub-periods. The analysis is carried out by using both standard parametric and nonparametric approaches. With the standard parametric approaches, the study uses the Wald test and the GRS test to examine if multivariate intercepts generated from the CAPM jointly equal zero. Results generated from both tests firmly reject the hypothesis of joint zero intercepts. This implies that asset returns are not mean-variance efficient. The results are consistent with those reported in the U.S. market by Black, Jensen and Scholes (1972), MacKinlay and Richardson (1991) and Chou and Zhou (2006) but are somewhat contradictory to earlier Australian results reported by Faff (1991) and Wood (1991).

As the Wald and GRS tests are based on the assumption of normal errors, it is important to test the robustness of conclusions drawn from these tests when the assumptions are violated. A nonparametric approach such as bootstrapping, which evaluates the distribution of a statistic based on random sampling, is used. This method has advantages over the proposed parametric approach because in this method, the errors are sampled from the actual residuals generated from regressions of the multivariate CAPM and therefore the results overcome problems of non-normalities of the errors. Using the bootstrap method, the study finds that the Wald test is significantly affected by the non-normal properties of the errors and therefore its bootstrapped results are not consistent with the results generated from the normality assumption of the error terms. Gibbons, Ross and Shanken (1989) point out that that the Wald-statistic is very sensitive to the distribution of the error term and therefore inferences drawn from it can be seriously misleading if the error term largely departs from normality. This perhaps can explain why the results generated from the Wald test are not consistent between the two cases of the error distributions. On the other hand, the results for the GRS test are consistent in both cases of the error distributions. This is because the GRS-statistic follows a non-central F-distribution while general F-tests are fairly robust even if the error distribution are not normal (MacKinlay 1985). Because of the superior performance of the GRS-statistic, the following chapters only use this methodology and not the Wald test.

As skewness and kurtosis are associated with non-normalities of asset returns while asset returns are not mean-variance efficient, this leads the study to investigate whether asset returns are mean-variance-skewness-kurtosis efficient. In the next chapters, the study investigates whether the four-moment model, which is the CAPM incorporating systematic skewness and systematic kurtosis, can explain expected returns in time series and cross-section.

CHAPTER 5. ARE ASSET RETURNS MEAN-VARIANCE-SKEWNESS-KURTOSIS EFFICIENT?

5.1 Introduction

The results reported in Chapter 4 suggest that mean and variance alone are not sufficient to characterise return behaviour. Given that the empirical stock return distribution is observed to be asymmetric and leptokurtic, a natural extension of the two-moment asset pricing model is to incorporate skewness and kurtosis as risk factors into the asset pricing models. This intuitive approach is motivated by Rubinstein (1973), who shows that measuring risk requires more than just covariance if returns do not follow a normal distribution. In this chapter, the study develops a four-moment model by incorporating systematic skewness and systematic kurtosis into the CAPM. This model will be used to examine whether asset returns are mean-variance-skewness-kurtosis efficient in the context of the Australian aggregate market and different industry sectors.

Merton (1973) suggests that if an asset-pricing model is well-specified, regressions generated from this model would produce intercepts that are indistinguishable from zero. Similarly to the methodology proposed in Chapter 4, the study in this chapter tests the validity of the four-moment model by examining the intercept or the pricing error of the model. The study applies the generalised multivariate method of the GRS test proposed in Chapter 4 to examine whether the proposed four-moment model is empirically valid, i.e. whether the pricing error approaches zero when systematic skewness and systematic kurtosis are included. This is equivalent to testing whether asset returns are mean-variance-skewness-kurtosis efficient.

Using portfolios formed on the basis of systematic skewness and systematic kurtosis characteristics, the study shows that the pricing error of the CAPM can be explained by systematic skewness and systematic kurtosis. Furthermore, the study fails to reject the null hypothesis of multivariate zero intercepts when the intercepts are generated from the multivariate four-moment model. This suggests that asset returns are mean-variance-skewness-kurtosis efficient and the four-moment model is adequate to explain variations of asset returns. The results are robust among the five sub-periods of 1992–1996, 1997–2001, 2002–2006 and 1992–2006, the last being the period before the global financial crises (GFC), but not the 2007–2009 period of GFC.

As mentioned in Chapter 4, the GRS test is based on the assumption that regression errors are normal. Although in Chapter 4 the study provides evidence that the GRS test for the CAPM is reasonably robust if the errors depart from normality, a robustness check will be carried out of whether inferences drawn from the generalised GRS test for the four-moment model in this chapter are consistent even if the assumption of normal errors is violated. Similarly to the methodology proposed in Chapter 4, the study in this chapter uses the bootstrap method for this robustness check. The bootstrap test concludes that the generalised GRS test is robust even when regression errors are non-normal.

In an attempt to examine the sensitivity of expected returns to the existence of the systematic skewness and systematic kurtosis in Australian data, this study uses the time-series regression approach of Fama and French (1992). The results provide strong evidence that systematic skewness and systematic kurtosis contribute significantly to the variation of time-series asset returns. The findings are consistent with Kraus and Litzenberger (1976), who reveal that the skewness factor can capture the variation in time-series returns which the market factor fails to explain. Furthermore, the study finds that the roles of these factors in explaining the variations in expected returns are more critical in bear periods than in bull periods.

The majority of studies on higher moments are on the U.S. market and little research has been done on the Australian stock market. This study on Australian stocks at both aggregate and industry levels makes a valuable contribution to the existing asset pricing literature. At the aggregate level, using skewness-based and kurtosis-based portfolios, the study finds that asset returns with high skewness and/or kurtosis risk generally earn higher risk premiums. It is also interesting to observe that for Australian stocks, the skewness effects are more important than the kurtosis effects. At the industry level, using industry-based portfolios, the study shows that cyclical sectors with more volatile cash flows and high leverage, such as the materials, industrials and information technology sectors, are very susceptible to market conditions and therefore to systematic skewness risk. On the other hand, growth sectors which rely more heavily on the present value of future growth opportunities, such as the industrials, telecommunication, property, consumer discretionary and health care sectors, are more vulnerable to external shocks and therefore to systematic kurtosis risk.

The chapter is organised as follows. The methodology is presented for testing the pricing error of the four-moment model by using skewness-based and kurtosis-based portfolios, and then for examining whether systematic skewness and systematic kurtosis can capture variations of asset returns by using industry-based portfolios. Empirical results and discussion follow and finally conclusions.

5.2 Methodology

In this section, the study presents a generalised multivariate regression approach to test the validity of the four-moment model. The study also presents a bootstrap method to check the robustness of results generated from the multivariate regressions. Details of these tests are as follows.

5.2.1 The Validity of Asset Pricing Models: A Multivariate Test of Time-Series Regressions

Merton (1973) suggests that if an asset-pricing model is well-specified, regressions generated from this model will produce intercepts that are indistinguishable from zero. In other words, if a proposed asset pricing model holds, there is no way to group assets into portfolios whose intercepts are reliably different from zero. Thus, one method to understand how systematic skewness and kurtosis enter asset pricing is to analyse the pricing error of the four-moment model. The study uses a multivariate approach to examine the pricing error. The advantage of the multivariate approach is that it does not require specifying a particular alternative hypothesis. Empirical studies such as the Shanken (1982, 1985 and 1992) and the Shanken and Zhou (2007) suggest that the multivariate approach can lead to more appropriate conclusions than those based on traditional inference, which relies on a set of dependent univariate statistics. This study applies the generalised GRS test to examine whether the proposed four-moment model is empirically valid, i.e. whether the pricing error approaches zero when systematic skewness and systematic kurtosis are included in the CAPM.

Consider the following multivariate linear regression model in the general form:

$$\tilde{r}_{it} = \alpha_{0i} + \sum_{j=1}^{L} \beta_{ij} \tilde{f}_{jt} + \tilde{\epsilon}_{it}, \qquad \forall i = 1,..,N \text{ and } \forall\, t = 1,..,T \quad (5.1)$$

where $\tilde{r}_{it}$ is the excess return on asset i in period t; $\tilde{f}_{jt}$ is the explanatory factor j in period t, $\tilde{\epsilon}_{it}$ is the disturbance term for asset i in period t; N is the number of underlying assets, T is the number of time-series observations and L is the number of regression parameters excluding the intercept. The disturbances are assumed to be jointly normally distributed in each period with mean zero and non-singular variance covariance matrix Σ. The disturbances are also assumed to be independent over time.

If the asset pricing model is efficient, then the following first-order condition must be satisfied for N assets:

$$E(\tilde{r}_{it}) = \sum_{j=1}^{L} \beta_{ij} E(\tilde{f}_{jt}), \qquad (5.2)$$

Combining the first-order condition in equation (5.2) with the distributional assumption given in equation (5.1) yields the following parameter restriction, which is stated in the form of a null hypothesis:

$$H_0: \alpha_{0i} = 0, \qquad \forall\, i = 1,..,N \qquad (5.3)$$

If the multivariate model in equation (5.1) is estimated using the Ordinary Least Squares (OLS) for each individual equation, the estimated intercepts have a multivariate normal distribution conditional on $\tilde{f}_{jt}$, with:

$$\sqrt{T/(1+\hat{\theta}^2)}\, \hat{\alpha}_{0i} \sim N\left\{\sqrt{T/(1+\hat{\theta}^2)}\, \alpha_{0i}; \Sigma\right\}, \qquad (5.4)$$

where $\hat{\theta} = \bar{r}/_{s}$ where $\bar{r}$ is the vector of sample mean of $\tilde{f}_{jt}$, s is the vector of sample standard deviation of $\tilde{f}_{jt}$ and $\hat{\Sigma}$ is the variance-covariance matrix of the errors.

The generalised GRS-statistic is computed to test whether the intercepts (α_{0i}) from OLS regressions are jointly equal to zero:

$$H_0\text{: } \alpha_{0i} = 0, \qquad \forall\ i = 1, .., N. \tag{5.5}$$

The generalised GRS-statistic is computed as

$$\text{GRS} - \text{statistic} = \left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left(1 + \bar{f}'\hat{\Omega}^{-1}\bar{f}\right)^{-1}\left(\hat{\alpha}_0{}'\hat{\Sigma}^{-1}\hat{\alpha}_0\right), \tag{5.6}$$

where $\bar{f}$ is the vector of sample mean of $\tilde{f}_t = \left(\tilde{f}_{1t}, \tilde{r}_{2t}.., \tilde{f}_{Lt}\right)$;

$\hat{\Omega}$ is the sample variance-covariance matrix of $\tilde{f}_t$; ;

$\hat{\Sigma}$ is the variance-covariance matrix of the residuals from the OLS regressions;

$\hat{\alpha}_0 = (\alpha_{01}, \alpha_{02}, .., \alpha_{0N})$ is the vector of the least squares estimators for the pricing error; where α_{0i} is the intercept of the regression of portfolio i on L regression parameters;

N is the number of underlying assets;

L be the number of regression parameters; and

T is the number of time-series observations.

In this study, the multivariate test of joint zero intercepts is applied for the four-moment model:

$$R_{it} - r_{ft} = \alpha_i + \beta_{1,i} r_{mt} + \beta_{2,i} S_t + \beta_{3,i} K_t + \varepsilon_{it}, \qquad \forall\ i = 1, .., N \tag{5.7}$$

where r_{mt} is the excess return of the market portfolio at time t, and S_t and K_t are the systematic skewness and kurtosis premiums.

The GRS-statistic has an F-distribution with degrees of freedom N and T-N-L. The GRS-statistic is equivalent to the usual t-statistic on the single intercept term in a univariate regression model.

5.2.2 The Validity of Asset Pricing Models: A Robustness Check Using the Bootstrap Method

The assumption that the error terms are normal is critical in statistical analysis. However, this assumption does not usually hold, so efficiency tests based on parametric approaches may fail to give satisfactory results. Although the results of the GRS test for the CAPM in Chapter 4 reveal that the test is reasonably robust with respect to typical levels of non-normalities found in the error terms for the two-moment model, it is important to check if the results generated from the generalised GRS test for the four-moment model in the previous section is robust if the error terms depart from normality. Similarly to the methodology outlined in Chapter 4, the bootstrap method is used to re-examine the efficiency of the four-moment model. The bootstrap efficiency test is designed as follows:

1. Estimate the multivariate four-moment model using the OLS method to obtain $\hat{\alpha}$, $\hat{\varepsilon}_t$, $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$:

$$R_t - r_{ft} = \hat{\alpha} + \hat{\beta}_1 r_{mt} + \hat{\beta}_2 S_t + \hat{\beta}_3 K_t + \hat{\varepsilon}_t. \quad (5.8)$$

2. Calculate the generalised GRS-statistic as specified in equation (5.6):

$$\text{GRS} - \text{statistic} = \left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left(1 + \bar{f}'\hat{\Omega}^{-1}\bar{f}\right)^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \ . \quad (5.9)$$

3. Repeat the following steps a large number of times (5000):

   a. Resample $\hat{\epsilon}_t^*$ (t=1,..,N) from $\{\hat{\varepsilon}_t\}_{t=1}^{T}$ achieved from step 1 with replacement.

   b. Generate simulated excess returns under the null hypothesis (intercept=0):

   $$r_t^* = \beta_1 r_{mt} + \beta_2 S_t + \beta_3 K_t + \hat{\epsilon}_t^*, \quad t = 1, .., T; \qquad (5.10)$$

   where $\beta_i = \hat{\beta}_i$; $\hat{\varepsilon}_t^*$ is generated from step a and $\hat{\beta}_i$ are generated from step 1.

   c. Regress r_t^* on r_{mt}, S_t and K_t using the OLS method to obtain estimates $\hat{\alpha}^*, \hat{\beta}_1^*, \hat{\beta}_2^*$ and $\hat{\beta}_3^*$ and $\hat{\Sigma}^*$. Recalculate the generalised GRS-statistic as follows:

   $$GRS - statistic^* = \left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left(1 + \bar{f}'\hat{\Omega}^{-1}\bar{f}\right)^{-1}\hat{\alpha}^{*\prime}\,\hat{\Sigma}^{*-1}\hat{\alpha}^* \quad (5.11)$$

4. Calculate the percentage of $GRS - statistic^*$ that is greater than $GRS - statistic$ measured in step 1. The percentage is the p-value of the bootstrap test.

5.2.3 Are Skewness and Kurtosis Important Asset Pricing Factors? A Time-series Regression Analysis

As the pricing error is expected to become insignificant when systematic skewness and systematic kurtosis are added to the two-moment model, there is a need to investigate in detail how these two factors explain patterns of average returns. While the previous section focuses on the significance of the pricing error of the four-moment model, this section focuses on examining the explanatory power of systematic skewness and systematic kurtosis on asset returns by using the Fama and French (1992) time-series regression approach. The four-moment model for time-series regression analysis is re-specified as:

$$R_t - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \beta_2 S_t + \beta_3 K_t + \varepsilon_t, \qquad (5.12)$$

where R_t is the portfolio return at time t, r_{ft} is the risk-free rate of return at time t, R_{mt} is the return of the market index at time t, and S_t and K_t are the systematic skewness and kurtosis premiums.

In this regression analysis, the explanatory variables are the returns of the market portfolio and the mimicking portfolios for the systematic skewness and systematic kurtosis factors in returns. The ASX300 index return minus the 90-day Bank Bill Accepted rate is used as a proxy for the market premium. The mimicking portfolios for systematic skewness and systematic kurtosis are formed to mimic the underlying risk factors in returns related to systematic skewness and systematic kurtosis. The returns of these mimicking portfolios are the differences between the returns of the highest systematic skewness (kurtosis) portfolio and the returns of the lowest systematic skewness (kurtosis) portfolio. The dependent variables are the excess returns on 25 portfolios, which are formed on the basis of both systematic skewness and systematic kurtosis as described in Chapter 3. The study uses dependent portfolios formed on the basis of both systematic skewness and systematic kurtosis because the study seeks to determine whether the returns of the mimicking portfolios used as explanatory variables can capture common factors in stock returns related to systematic skewness and systematic kurtosis.

Using equation (5.12), weekly portfolio returns are regressed on the returns of the market portfolios and the mimicking portfolios for systematic skewness and systematic kurtosis. The time-series regression slopes, β_i, are factor loadings that are interpreted as risk factor sensitivities for stock returns. They present the average premium per unit of risk for the candidate common risk factors in returns. Equation (5.12) indicates that if the factor loadings are significant, the market premium, the systematic skewness and the systematic kurtosis factors are correspondingly proxies for sensitivities to common risk factors in asset returns. Overall, the slopes and adjusted R-squared values of the model show how mimicking portfolios for risk factors related to systematic skewness and systematic kurtosis capture the variations in asset returns.

5.3 Results and Discussion

In this section, the study presents the regression results for the multivariate tests outlined in the methodology, the results for the robustness check using the bootstrap method and the time-series regression results for the four-moment model. Details are as follows.

5.3.1 The Validity of Asset Pricing Models: Multivariate Tests of Zero Intercepts

To understand how systematic skewness and systematic kurtosis influence asset pricing, this study employs a multivariate approach. In particular, the study analyses the pricing error of the four-moment model. The generalised Gibbons, Ross and Shanken (1989) multivariate test of joint zero intercepts, which are generated from the time-series regressions of the portfolio returns on the risk factors, is performed. The main idea of this test is that if the intercepts from the regressions are jointly equal to zero, i.e. the GRS-statistic is not statistically significant, the risk factors are sufficient to explain the variation in asset returns. However, if the intercepts are jointly different to zero, i.e. the GRS-statistic is statistically significant, then the model is not effective in explaining the variation in asset returns. If the GRS-statistic decreases consistently when more risk factors are incorporated into the model, these factors do add value to the CAPM.

Table 5.1 Multivariate Tests of Joint Zero Intercepts of Four-Moment Model

The table reports statistics of the multivariate tests on joint zero intercepts of time-series regressions of the two-moment and four-moment models. The two-moment and four-moment models are specified as follows:

The two-moment model: $R_t - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \varepsilon_t$;

The four-moment model: $R_t - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \beta_2 S_t + \beta_3 K_t + \varepsilon_t$;

where R_t is the return of the portfolio at time t, r_{ft} is the risk-free rate of return at time t, R_{mt} is the return of the market index at time t, S_t and K_t are the systematic skewness and kurtosis factors. The GRS-statistic follows an F-distribution with degrees of freedom N and T-N-L where T is the total observations, N is the number of portfolios and L is the number of regression parameters in the model. P-values are presented in parentheses. * and ** denote statistical significance at 5 and 1 percent levels.

Period	Two-moment model		Four-Moment Model	
	Normal errors	Non-normal errors	Normal errors	Non-normal errors
	GRS-statistic	Bootstrapped P-value	GRS-statistic	Bootstrapped P-value
1992-1996 (P-value)	1.512 (0.061)	0.069	0.905 (0.598)	0.701
1997-2001	1.650 (0.031)*	0.048*	1.125 (0.315)	0.364
2002-2006	1.703 (0.023)*	0.037*	1.446 (0.063)	0.083
2007-2009	2.454 (0.001)**	0.018*	2.108 (0.005)**	0.031*
1992-2006	1.727 (0.015)**	0.032*	1.415 (0.091)	0.1484
whole period	1.896 (0.005)**	0.013*	1.729 (0.014)*	0.035*

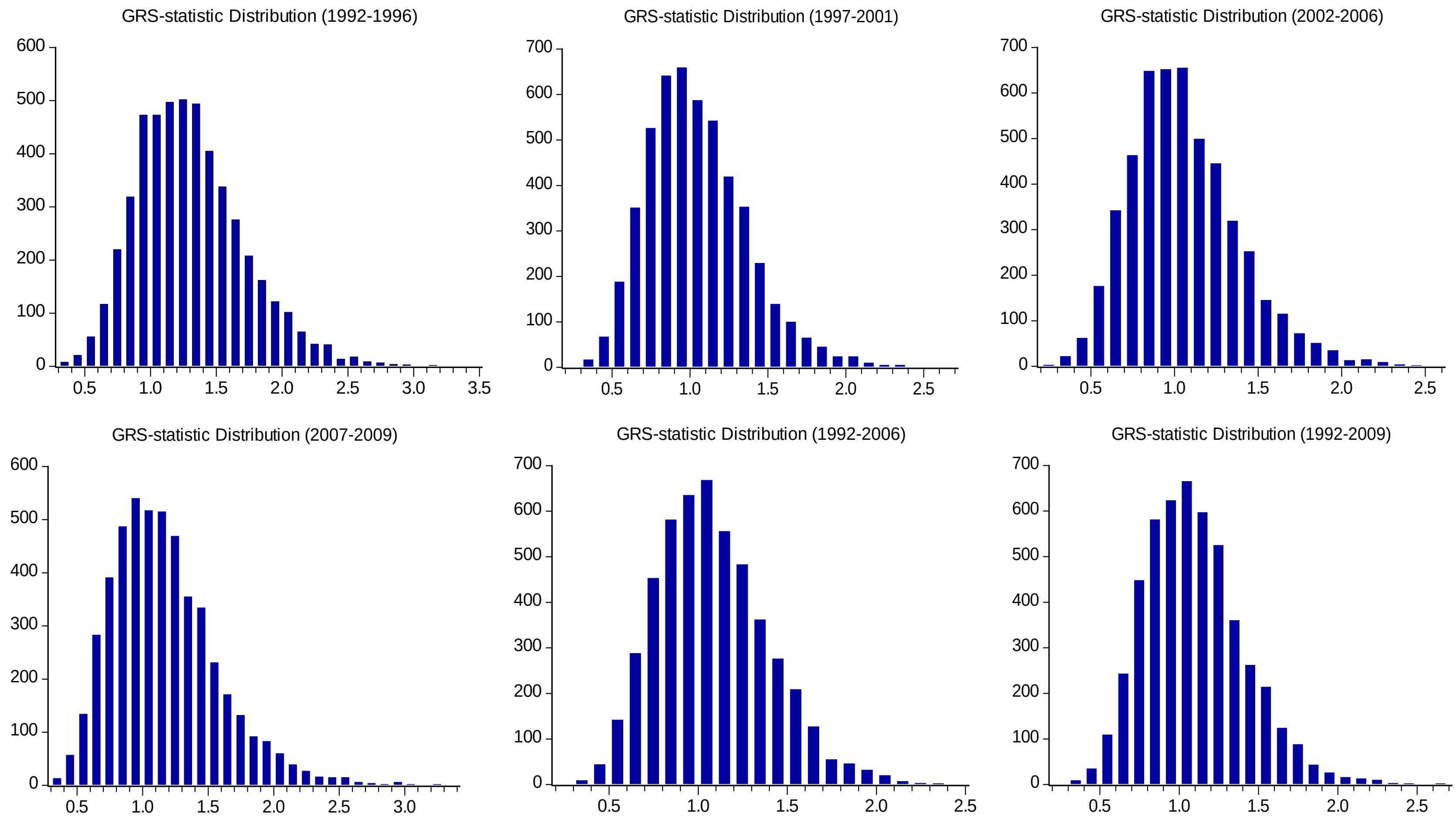
Table 5.1 provides the results of the GRS tests for the two-moment and four-moment models. The results from the GRS test for the two-moment CAPM are taken from Chapter 4 to allow comparisons with those generated from the four-moment model. To check the robustness of these results, the tests of zero joint intercepts are also performed for five sub-periods: 1992–

1996, 1997–2001, 2002–2006, 2007–2009 and 1992–2006. As concluded in Chapter 4, the GRS-statistic for the two-moment model shows that asset returns are not mean-variance efficient and the market beta alone is not sufficient to explain asset returns. As a result, the study now tests if additional explanatory variables such as systematic skewness and systematic kurtosis are able to explain patterns in the asset returns that are not explained by the market factor. It is observed that the p-values for the GRS-statistic increase for every period examined when the extra variables are included. In three of the four sub-periods where it was significant at the 5 percent level, it is now not significant. This shows that systematic skewness and systematic kurtosis are more successful at explaining returns in all periods and the four-moment model seems to be efficient in at least three sub-periods and for the whole period excluding the GFC.

Table 5.1 also shows that the null hypothesis of joint zero intercepts for the four-moment model is not rejected at the 5 percent level in four out of the five sub-periods examined, except for the 2007–2009 period. Although the hypothesis is rejected for the entire period of 1992–2009, it is not rejected for the 1992–2006 period, which is before the GFC. It is suggested that the 2007–2009 period of the GFC creates extensive exogenous shocks to the global market performance and therefore biases most of the fundamental analyses. Nevertheless, the overall evidence supports the validity of the four-moment model in most conditions as the period from 1992 to 2006 includes the Asian Financial Crisis in 1997, the Dot-com Bubble Deflation in 2001 and the September 11th 2001 event. The finding also emphasises the importance of systematic skewness and systematic kurtosis as explanatory variables for asset returns that are not explained by the market factor.

Figure 5.1 Bootstrap Distributions of GRS-statistics

The figure presents bootstrapped distributions of GRS-statistics for the period of 1992–2009 and five sub-periods: 1992–1996, 1997–2001, 2002–2006, 2007–2009 and 1992–2006. The GRS-statistic sample is generated from the bootstrap method with the number of repetitions of 5000. The GRS-statistic is calculated as: $\mathrm{GRS-statistic} = \left(\frac{T}{N}\right)\left(\frac{T-N-1}{T-L-1}\right)\left(\bar{f}'\widehat{\Omega}^{-1}\bar{f}\right)^{-1}\widehat{\alpha}'\widehat{\Sigma}^{-1}\widehat{\alpha}$ where $\mathrm{GRS-statistic} \sim F(N, T-N-L)$; $\bar{f}$ is the sample mean of f_t where $f_t = (f_{1t}, \ldots, f_{Lt})'$; $\widehat{\Omega}$ is the sample variance-covariance matrix for f_t, $\widehat{\Sigma}$ is the variance-covariance matrix of the error terms, T is the time-series observations, N is the number of portfolios and L is the number of regression parameters in the model.



Using the bootstrap method to check the robustness of the generalised GRS test for the four-moment model, the study concludes that the generalised GRS test is robust even if the regression errors are non-normal. Figure 5.1 presents the empirical distributions of the GRS statistics generated by the bootstrap method with 5000 repetitions. Table 5.1 shows that the conclusions drawn from the generalised GRS test for the four-moment model in the case of normal errors are consistent with those drawn in the case of non-normal errors.

5.3.2 Can Systematic Skewness and Systematic Kurtosis Explain Patterns of Expected Returns? A Time-series Regression Analysis

In this section, a time-series regression analysis is conducted to examine the power of systematic skewness and systematic kurtosis in explaining the variation of time-series asset returns. Weekly portfolio returns are regressed on the returns of the market portfolio and the mimicking portfolios for systematic skewness and systematic kurtosis. The average excess returns on the 25 portfolios that serve as dependent variables give perspective to the range of average returns that competing sets of risk factors must explain. The average returns on the explanatory portfolios are the average premium per unit of risk for the candidate common risk factors in returns. The slopes and adjusted R-squared values of the regressions show whether the mimicking portfolios for risk factors based on systematic skewness and systematic kurtosis capture the variation in asset returns.

Table 5.2 reports regression results for weekly Australian stocks for the period from 1992 to 2009. As expected, the market premium coefficient is significant in every regression. The result substantially supports the well-known findings of Markowitz (1952), Sharpe (1964) and Lintner (1965) that the market factor is important in explaining patterns of asset returns. Overall, 24 out of the 25 regressions have systematic skewness or systematic kurtosis or both significant at the 5 percent level. This signifies that systematic skewness and systematic kurtosis factors do contribute significantly to the variation in asset returns. The systematic skewness coefficients are positive for low skewness portfolios and negative for high skewness portfolios. This is consistent with the observations by Arditti (1967) and Ingersoll (1975) that risk-averse investors have to forgo the expected portfolio return if they want to gain benefit from positive portfolio skewness and vice versa. This also represents a negative trade-off between asset returns and skewness as investors require more return premium to hold negative skewness portfolios while they are willing to receive less premium or even pay a premium to hold high positive skewness portfolios. It is observed that the kurtosis coefficients are negative for low kurtosis portfolios and positive for high kurtosis portfolios. The results also suggest that the kurtosis premium increases when stocks are more exposed to kurtosis risk.

Overall, the regression results in table 5.2 show that systematic skewness and systematic kurtosis are important for explaining patterns of time-series asset returns. The results are consistent with Scott and Horvath (1980), Hwang and Satchell (1999), Galagedera, Henry and Silvapulle (2003) and Jondeau and Rockinger (2006), who argue that investors have a negative preference for even moments (i.e. variance and kurtosis) and a positive preference for odd moments (i.e. return and skewness). Consequently, there are negative trade-offs between the asset returns and systematic skewness and positive trade-offs between the returns and systematic kurtosis.

Table 5.2 Regression results of 25 portfolios formed by systematic skewness and systematic kurtosis: January 1992 to May 2009

The table presents regressions results with Newey-West standard errors for the four-moment model: $R_{it} - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \beta_2 S_t + \beta_3 K_t$ where R_{it} is the return of portfolio i at time t, r_{ft} is the risk-free rate of return at time t, R_{mt} is the return of the market index at time t, S_t and K_t are the return premium for the systematic skewness and systematic kurtosis factors respectively. Student's t-statistics are reported in the parentheses below the coefficient estimates. * and ** denote statistical significance at 5 and 1 percent levels.

Panel A	**Systematic Skewness (β_2)**					**Systematic Kurtosis (β_3)**				
	Low Sys Kurtosis	**2**	**3**	**4**	**High Sys. Kurtosis**	**Low Sys. Kurtosis**	**2**	**3**	**4**	**High Sys. Kurtosis**
Low Sys. Skewness	0.0375 (-0.76)	0.1350 (1.72)	0.3014 (3.37)**	0.7128 (2.17)*	3.1038 (2.51)**	-0.2323 (-4.66)**	0.0131 (0.17)	0.4159 (4.74)**	0.8121 (3.13)**	3.6388 (2.94)**
2	-0.2979 (-5.03)*	-0.1242 (-2.50)**	-0.0468 (-0.75)	0.0866 (0.89)	0.3963 (2.65)**	-0.3924 (-6.57)**	-0.1639 (-3.11)**	0.1389 (2.11)*	0.3117 (3.38)**	0.9526 (6.81)**
3	-0.7102 (-3.84)**	-0.3065 (-6.41)**	-0.2649 (-4.88)**	-0.0577 (-0.86)	0.2990 (3.45)**	-0.6728 (-4.10)**	-0.2495 (-5.13)**	-0.0318 (-0.53)	0.2741 (4.38)**	0.9787 (11.50)**
4	-1.2408 (-6.89)**	-0.6298 (-6.65)**	-0.5844 (-7.42)**	-0.3200 (-5.16)**	0.1086 (2.03)*	-0.8642 (-5.32)**	-0.2543 (-2.68)**	-0.0724 (-0.79)	0.1333 (2.19)*	0.7707 (15.01)**
High Sys. Skewness	-1.5928 (-10.24)**	-1.4555 (-8.54)**	-1.1241 (-12.80)**	-1.0800 (-17.93)**	-0.6337 (-12.29)**	-1.1164 (-7.35)**	-0.8939 (-6.21)**	-0.5231 (-6.71)*	-0.2813 (-4.21)**	0.4137 (8.53)**

Panel B	**Market Premium (β_1)**					**Adjusted R-squared**				
	Low Sys. Kurtosis	**2**	**3**	**4**	**High Sys. Kurtosis**	**Low Sys Kurtosis**	**2**	**3**	**4**	**High Sys. Kurtosis**
Low Sys. Skewness	0.6279 (19.18)**	0.6773 (19.96)**	0.7316 (14.82)**	0.6840 (4.92)**	0.4581 (1.99)*	0.549	0.543	0.407	0.194	0.113
2	0.6197 (18.03)**	0.6919 (21.66)**	0.6936 (15.99)**	0.7185 (17.50)**	0.6284 (10.10)**	0.534	0.610	0.492	0.509	0.390
3	0.5829 (7.85)**	0.6614 (18.86)**	0.6722 (19.00)**	0.6543 (17.34)**	0.6188 (10.79)**	0.113	0.543	0.610	0.596	0.524
4	0.5879 (6.58)**	0.5663 (10.10)**	0.6269 (11.19)**	0.6899 (20.17)**	0.6213 (18.40)**	0.119	0.260	0.487	0.623	0.746
High Sys. Skewness	0.5856 (7.16)**	0.6862 (7.61)**	0.5906 (11.34)**	0.6859 (17.93)**	0.6351 (19.41)**	0.209	0.180	0.394	0.704	0.848

Table 5.3 Regression results of 25 portfolios formed by systematic skewness and systematic kurtosis for the period of 1992–1996

The table presents regressions results with Newey-West standard errors for the four-moment model: $R_{it} - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \beta_2 S_t + \beta_3 K_t$ where R_{it} is the return of portfolio i at time t, r_{ft} is the risk-free rate of return at time t, R_{mt} is the return of the market index at time t, S_t and K_t are the systematic skewness and systematic kurtosis factors. Student's t-statistics are reported in the parentheses below the coefficient estimates. * and ** denote statistical significance at 5 and 1 percent levels.

Panel A

	Systematic Skewness (β_2)					Systematic Kurtosis (β_3)				
	Low S. Kurtosis	2	3	4	High S. Kurtosis	Low S. Kurtosis	2	3	4	High S. Kurtosis
Low S. Skewness	0.4214 (4.92)**	0.3622 (3.99)**	0.3517 (3.93)**	0.8227 (9.52)**	1.0569 (7.60)**	-0.5152 (-6.15)**	0.0661 (0.80)	-0.0176 (-0.20)	0.1554 (1.82)	0.6161 (5.35)**
2	0.2134 (3.49)**	0.1000 (1.72)	0.2010 (1.98)*	0.3172 (2.28)*	0.4435 (2.45)**	-0.3008 (-4.67)**	-0.0613 (-1.04)	-0.0939 (-1.65)	-0.0327 (-0.51)	0.3019 (2.39)**
3	-0.0094 (-0.08)	0.1542 (2.13)*	0.0795 (1.33)	0.1577 (2.02)*	0.2002 (2.18)*	-0.6991 (-6.22)**	-0.1868 (-2.91)*	-0.0937 (-1.78)	-0.2109 (-3.44)**	0.1643 (2.02)*
4	-0.0730 (-0.87)	0.1197 (1.50)	0.0317 (0.56)	0.0728 (1.46)	0.1744 (2.00)**	-0.6665 (-7.16)**	-0.2511 (-3.29)**	-0.0982 (-1.67)	-0.0494 (-0.91)	0.2293 (3.33)**
High S. Skewness	-0.5628 (-3.87)**	-0.3291 (-2.38)*	-0.2177 (-1.96)*	-0.0581 (-0.78)	-0.1061 (-1.46)	-0.9597 (-8.55)**	-0.2122 (-2.26)**	-0.1788 (-2.09)*	-0.0485 (-0.68)	0.3105 (4.55)**

Panel B

	Market Premium (β_1)					Adjusted R-squared				
	Low S. Kurtosis	2	3	4	High S. Kurtosis	Low S. Kurtosis	2	3	4	High S. Kurtosis
Low S. Skewness	0.8380 (14.26)**	0.8122 (13.55)**	0.7718 (10.60)**	0.7371 (9.93)**	0.8800 (10.38)**	0.514	0.543	0.483	0.541	0.592
2	0.7160 (13.66)**	0.7647 (17.52)**	0.7578 (16.25)**	0.8406 (18.49)**	0.8092 (11.81)**	0.594	0.643	0.653	0.630	0.360
3	0.8040 (8.36)**	0.7348 (16.20)**	0.7541 (19.57)**	0.8875 (16.11)**	0.8736 (15.63)**	0.323	0.623	0.633	0.611	0.522
4	0.7948 (10.25)**	0.6992 (9.19)**	0.7186 (15.99)**	0.8079 (20.51)**	0.8308 (18.20)**	0.369	0.334	0.598	0.700	0.683
High S. Skewness	0.9570 (11.92)**	0.7769 (11.36)**	0.7069 (9.21)**	0.8040 (15.62)**	0.7804 (16.91)**	0.407	0.436	0.397	0.589	0.679

Table 5.4 Regression results of 25 portfolios formed by systematic skewness and systematic kurtosis for the period of 1997–2001

The table presents regressions results with Newey-West standard errors for the four-moment model: $R_{it} - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \beta_2 S_t + \beta_3 K_t$ where R_{it} is the return of portfolio i at time t, r_{ft} is the risk-free rate of return at time t, R_{mt} is the return of the market index at time t, S_t and K_t are the systematic skewness and systematic kurtosis factors. Student's t-statistics are reported in the parentheses below the coefficient estimates. * and ** denote statistical significance at 5 and 1 percent levels.

Panel A	Systematic Skewness (β_2)					Systematic Kurtosis (β_3)				
	Low S. Kurtosis	2	3	4	High S. Kurtosis	Low S. Kurtosis	2	3	4	High S. Kurtosis
Low S. Skewnes	0.3969 (1.95)*	0.5934 (5.96)**	1.0258 (4.23)**	1.1325 (6.33)**	0.8209 (3.13)**	0.1643 (1.32)	0.4615 (5.74)**	0.9941 (4.96)**	1.2598 (8.47)**	1.4475 (6.54)**
2	-0.2249 (-2.57)**	-0.0416 (-0.54)	0.2105 (1.98)*	0.5832 (3.19)**	0.8339 (1.65)	-0.2861 (-4.14)**	-0.0966 (-1.59)	0.2202 (2.57)**	0.8295 (5.77)**	1.4564 (3.31)**
3	-0.6659 (-3.42)**	-0.2018 (-2.21)*	-0.0612 (-0.71)	0.2168 (1.67)	0.9983 (4.40)**	-0.6403 (-3.40)**	-0.1405 (-2.11)*	0.1962 (2.28)*	0.4809 (4.87)**	1.4760 (7.99)**
4	-0.6191 (-2.11)*	-0.3649 (-1.74)	-0.2346 (-2.05)*	0.0279 (0.22)	0.5356 (3.19)**	-0.3900 (-1.47)	-0.0050 (-0.03)	0.1501 (1.99)*	0.3936 (4.14)**	1.0688 (8.31)**
High S. Skewnes	-0.8820 (-1.99)*	-0.7981 (-3.31)**	-0.5636 (-3.59)**	-0.4206 (-2.56)**	-0.3572 (-1.96)*	-0.6641 (-2.67)**	-0.5163 (-2.37)**	-0.1142 (-0.99)	0.2447 (2.15)*	0.7078 (6.44)**

Panel B	Market Premium (β_1)					Adjusted R-squared				
	Low S. Kurtosis	2	3	4	High S. Kurtosis	Low S. Kurtosis	2	3	4	High S. Kurtosis
Low S. Skewness	0.5555 (8.22)**	0.6103 (10.55)**	0.6516 (8.81)**	0.6281 (6.86)**	0.3484 (2.19)*	0.388	0.566	0.559	0.508	0.358
2	0.6099 (12.23)**	0.6998 (14.73)**	0.7655 (12.48)**	0.7049 (8.19)**	0.4567 (3.07)**	0.538	0.650	0.613	0.547	0.231
3	0.5791 (5.52)**	0.7064 (14.08)**	0.7576 (14.77)**	0.6665 (10.12)**	0.7373 (6.97)**	0.152	0.616	0.703	0.644	0.629
4	0.5764 (3.43)**	0.6635 (6.61)**	0.6349 (9.75)**	0.6296 (9.92)**	0.6058 (9.50)**	0.095	0.336	0.634	0.718	0.770
High S. Skewness	0.8493 (6.12)**	0.6972 (7.00)**	0.5299 (6.05)**	0.6076 (9.29)**	0.5768 (8.44)**	0.210	0.261	0.412	0.681	0.870

Table 5.5 Regression results of 25 portfolios formed by systematic skewness and systematic kurtosis for the period of 2002–2006

The table presents regressions results with Newey-West standard errors for the four-moment model: $R_{it} - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \beta_2 S_t + \beta_3 K_t$ where R_{it} is the return of portfolio i at time t, r_{ft} is the risk-free rate of return at time t, R_{mt} is the return of the market index at time t, S_t and K_t are the systematic skewness and systematic kurtosis factors. Student's t-statistics are reported in the parentheses below the coefficient estimates. * and ** denote statistical significance at 5 and 1 percent levels.

Panel A

	Systematic Skewness (β_2)					Systematic Kurtosis (β_3)				
	Low S. Kurtosis	**2**	**3**	**4**	**High S. Kurtosis**	**Low S. Kurtosis**	**2**	**3**	**4**	**High S. Kurtosis**
Low S. Skewness	0.0700 (1.36)	0.2049 (4.17)**	0.2531 (3.44)**	0.1707 (1.73)	0.3992 (3.35)**	-0.1307 (-2.24)*	0.0747 (1.44)	0.2713 (3.48)**	0.3021 (2.96)**	0.6757 (5.51)**
2	-0.1324 (-4.32)**	-0.0436 (-1.36)	-0.0258 (-0.50)	0.0278 (0.35)	0.2217 (2.31)*	-0.1795 (-4.55)**	-0.0690 (-1.87)	0.0685 (1.30)	0.2718 (3.36)**	0.6218 (6.52)**
3	-0.3474 (-5.32)**	-0.1578 (-3.37)**	-0.1140 (-2.66)**	-0.0036 (-0.06)	0.0172 (0.24)	-0.2383 (-3.44)**	-0.0598 (-1.14)	0.0130 (0.29)	0.2125 (3.27)**	0.5091 (6.26)**
4	-0.3687 (-2.82)**	-0.3043 (-3.68)**	-0.3642 (-6.63)**	-0.2196 (-4.07)**	-0.1452 (-2.03)*	-0.1919 (-1.55)	-0.0537 (-0.70)	0.1111 (1.83)	0.1994 (3.47)**	0.4512 (5.64)**
High S. Skewness	-1.3070 (-6.42)**	-0.7816 (-3.92)**	-0.6923 (-9.40)**	-0.4642 (-8.73)**	-0.4999 (-8.56)**	-1.0858 (-5.29)**	-0.3888 (-2.10)*	-0.2241 (-2.82)**	0.0698 (1.16)	0.4556 (6.81)**

Panel B

	Market Premium (β_1)					Adjusted R-squared				
	Low S. Kurtosis	**2**	**3**	**4**	**High S. Kurtosis**	**Low S. Kurtosis**	**2**	**3**	**4**	**High S. Kurtosis**
Low S. Skewness	0.4717 (5.69)**	0.5008 (7.43)**	0.4725 (4.75)**	0.5270 (4.43)**	0.4644 (3.19)**	0.183	0.298	0.293	0.232	0.284
2	0.4408 (6.46)**	0.4574 (7.69)**	0.5173 (7.57)**	0.6224 (7.01)**	0.4866 (3.85)**	0.265	0.363	0.378	0.354	0.364
3	0.3885 (3.50)**	0.5067 (6.38)**	0.5355 (7.95)**	0.5472 (6.82)**	0.6003 (5.32)**	0.112	0.324	0.434	0.457	0.441
4	0.2491 (1.62)	0.4512 (4.19)**	0.3903 (4.33)**	0.4908 (6.49)**	0.5034 (5.73)**	0.045	0.201	0.435	0.555	0.624
High S. Skewness	0.6952 (4.85)**	0.5454 (3.05)**	0.5425 (5.42)**	0.4838 (5.61)**	0.4332 (4.77)**	0.411	0.150	0.409	0.519	0.745

Table 5.6 Regression results of 25 portfolios formed by systematic skewness and systematic kurtosis for the period of 2007-2009

The table presents regressions results with Newey-West standard errors for the four-moment model: $R_{it} - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \beta_2 S_t + \beta_3 K_t$ where R_{it} is the return of portfolio i at time t, r_{ft} is the risk-free rate of return at time t, R_{mt} is the return of the market index at time t, S_t and K_t are the systematic skewness and systematic kurtosis factors. Student's t-statistics are reported in the parentheses below the coefficient estimates. * and ** denote statistical significance at 5 and 1 percent levels.

Panel A — **Systematic Skewness (β_2)** / **Systematic Kurtosis (β_3)**

	Low S. Kurtosis	2	3	4	High S. Kurtosis	Low S. Kurtosis	2	3	4	High S. Kurtosis
Low S. Skewnes	0.2408 (3.34)**	0.2456 (3.78)**	0.4154 (3.62)**	0.5768 (5.89)**	0.7854 (7.13)**	-0.3191 (-5.84)**	-0.1654 (-3.36)**	0.1535 (2.03)*	0.3944 (5.44)**	0.8905 (11.34)**
2	0.0428 (0.77)	0.0571 (1.06)	0.1693 (2.40)**	0.2508 (2.29)*	0.4336 (4.03)**	-0.3313 (-7.54)**	-0.2460 (-6.40)**	-0.0066 (-0.13)	0.2860 (3.77)**	0.6951 (9.26)**
3	-0.1079 (-0.97)	-0.0445 (-0.58)	0.1187 (0.23)	0.1644 (1.97)*	0.2647 (2.40)**	-0.3115 (-3.98)**	-0.2113 (-3.87)**	-0.0417 (-0.70)	0.2566 (4.27)**	0.7105 (8.89)**
4	-0.3879 (-2.13)*	-0.2774 (-1.99)*	-0.1563 (-1.66)	-0.0899 (-1.12)	0.1008 (0.93)	-0.3712 (-3.13)**	-0.1605 (-2.20)*	-0.0602 (-0.84)	0.1585 (2.71)**	0.6347 (8.17)**
High S. Skewnes	-0.7128 (-3.31)**	-0.5911 (-4.68)**	-0.4986 (-4.05)**	-0.4150 (-4.10)**	-0.3637 (-3.96)**	-0.5258 (-3.31)**	-0.2759 (-3.06)**	-0.1243 (-1.40)	0.0787 (1.09)	0.5253 (7.30)**

Panel B — **Market Premium (β_1)** / **Adjusted R-squared**

	Low S. Kurtosis	2	3	4	High S. Kurtosis	Low S. Kurtosis	2	3	4	High S. Kurtosis
Low S. Skewness	0.6745 (9.95)**	0.7020 (11.20)**	0.7890 (10.67)**	0.6437 (7.64)**	0.5570 (5.75)**	0.624	0.613	0.653	0.730	0.851
2	0.6622 (9.08)**	0.7395 (13.20)**	0.7358 (11.20)**	0.6856 (7.16)**	0.6051 (6.67)**	0.661	0.698	0.672	0.663	0.760
3	0.6111 (5.54)**	0.7420 (9.75)**	0.7069 (11.35)**	0.6838 (8.80)**	0.6040 (6.32)**	0.362	0.646	0.654	0.705	0.798
4	0.5966 (5.36)**	0.6455 (7.91)**	0.6994 (9.00)**	0.7306 (11.18)**	0.6625 (8.42)**	0.285	0.454	0.563	0.734	0.839
High S. Skewness	0.5272 (4.15)**	0.7568 (6.93)**	0.7474 (8.80)**	0.7271 (8.85)**	0.7002 (9.11)**	0.408	0.467	0.626	0.688	0.825

5.3.3 Time-Series Analysis of Sub-periods

In this section, the study investigates how the impacts of systematic skewness and systematic kurtosis on asset pricing are differentiated by the economic cycle. The study examines these impacts in four different periods: 1992–1996, 1997–2001, 2002–2006 and 2007–2009. The periods of 1992–1996 and 2002–2006 are considered bull periods and the periods of 1997–2001 and 2007–2009 are considered bear periods.

Tables 5.3, 5.4, 5.5 and 5.6 provide regression results for the four sub-periods. The results show that the market factor is positive and statistically significant at the 1 percent level in every portfolio for all sub-periods. The factor loadings for systematic skewness and systematic kurtosis are generally significant at the 5 percent level in more than 60% of the regressions for every period examined. The results reinforce the finding in the multivariate test that the CAPM does not hold as it fails to predict that variables other than the market beta may explain the variation in asset returns. Consistent with the conclusion for the 1992–2009 period, the premium for skewness risk is positive for low skewness portfolios and negative for high skewness portfolios. The result is consistent with Arditti (1967), who proposes that risk-averse investors have to forgo their expected portfolio return if they want to gain more benefit from increasing portfolio skewness. Conversely, the premium for kurtosis risk is positive for high kurtosis portfolios and negative for low kurtosis portfolios. It is observed that controlling for skewness effects, the factor loading of systematic kurtosis tends to increase from low kurtosis to high kurtosis portfolios. Conversely, controlling for kurtosis effects, the factor loading of systematic skewness tends to decrease from low skewness to high skewness portfolios. The regression results also suggest that the effects of systematic skewness and systematic kurtosis are more prominent in bear periods than in bull periods. This is evidenced by more portfolios having significant systematic skewness and systematic kurtosis factor loadings in the 1997–2001 and 2007–2009 periods than in the 1992–1996 and 2002–2006 periods. Finally, the results lend support to Campbell and Hentschel (1992), who argue that investors are in favour of odd moments (i.e. return and skewness) but are averse to even moments of the expected returns (i.e. variance and kurtosis).

5.3.4 Sector Analysis

At the aggregate level, the study successfully shows that systematic skewness and kurtosis factors are important in explaining the patterns of the expected returns. In this section, the study uses industry-based portfolios to examine whether these factors are also relevant in explaining the variation of asset returns at the industry level. The study focuses on the sector index returns of 11 domestic sectors in Australia, namely, consumer discretionary, consumer staples, energy, financials excluding property trusts, health care, industrials, information technology, materials, property investment trusts, telecommunication services and utilities.

Table 5.7 presents descriptive statistics of 11 sector index returns. Utilities, materials and energy sectors are those with the highest mean returns and utilities, energy and telecommunication services sectors are those with the highest median returns. The industrials, properties investment trusts and consumer staples sectors have the lowest mean returns and the financials excluding property trusts, industrials and property investment trusts sectors have the lowest median returns. The index returns of the materials, consumer discretionary and financials excluding property trusts sectors are the most volatile while the return performances are most stable for the consumer staples, telecommunications and industrials sectors. Most of the sector index returns are slightly negatively distributed, except for financials and health care sectors, while all sector returns exhibit leptokurtic distributions. Finally, the Jarque-Bera test is presented to investigate the normality assumption of asset returns. The rejection of the normality hypothesis in every sector reinforces the suggestions of several studies such as Harvey and Zhou (1993) and Richardson and Smith (1993) that stock returns do not conform to a normal distribution.

Table 5.8 presents correlations between the sector index returns and between the sector index returns and the market index returns. Overall, the market index returns have moderate to strong correlations with the sectors index returns, ranging from 0.324 to 0.577. The consumer staples, telecommunication and property investment trusts sectors are those having the strongest correlations with the market returns. These sectors are also those with the lowest standard deviations, as evidenced in table 5.7. The materials, consumer discretionary and financials excluding property trusts sectors have the lowest correlations with the market returns. They are also those with highest standard deviations. Telecommunication is the only sector which has the majority of correlations with other sectors above 0.6. Financials excluding property trusts and materials are those with the lowest correlations with other sectors. Overall, the sector performances for Australian stocks are moderately correlated to each other and relatively strongly correlated to market performance.

Table 5.7 Descriptive statistics of sector index returns for the period of January 1992–May 2009

The table reports summary statistics of 11 sector index returns for the period of January, 1992 to May 2009. Mean and standard deviation are the first two moments of the return distribution while standardised skewness and kurtosis are the third and the fourth. Excess kurtosis is equal to the kurtosis of the portfolio minus 3, where 3 is the standardised kurtosis of the normal distribution. The total number of time-series observations is 906. The last column shows the number of firms in each sector.

	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Excess Kurtosis	Jarque-Bera	Probability	Number of firms
Consumer Discretionary	0.09%	0.14%	15.44%	-13.79%	3.36%	-0.028	1.92	140	0.00	38
Consumer Staples	0.10%	0.17%	11.27%	-8.43%	1.91%	-0.071	2.12	171	0.00	20
Energy	0.18%	0.30%	17.52%	-16.69%	3.04%	-0.393	3.13	393	0.00	36
Financials-exc. Property Trusts	0.15%	0.00%	30.17%	-16.77%	3.21%	0.757	11.16	4785	0.00	38
Heath Care	0.16%	0.24%	12.36%	-10.44%	2.66%	0.009	2.39	215	0.00	29
Industrials	-0. 03%	0.03%	17.43%	-24.21%	2.39%	-0.766	21.47	17462	0.00	51
Information Technology	0. 15%	0.24%	17.24%	-18.65%	3.00%	-0.518	3.91	616	0.00	30
Materials	0. 19%	0.10%	20.02%	-29.77%	4.26%	-0.463	4.90	936	0.00	78
Property Investment Trusts	0.06%	0.09%	9.28%	-13.49%	2.51%	-0.317	1.47	97	0.00	37
Telecommunication Services	0. 13%	0.30%	11.49%	-14.36%	2.50%	-0.588	4.48	810	0.00	7
Utilities	0. 21%	0.31%	16.60%	-20.29%	2.89%	-0.402	1.32	730	0.00	12

Table 5.8 Correlations between sector index returns for the period of January 1992–May 2009

The table presents correlation coefficients between 11 sector index returns and between the sector index returns and the market index returns.

	Consumer Discretionary	Consumer Staples	Energy	Financials-exc. Property Trusts	Heath Care	Industrials	Information Technology	Materials	Property Investment Trusts	Telecommunication Services	Utilities
Consumer Staples	0.573										
Energy	0.488	0.597									
Financials-exc. Property Trusts	0.407	0.499	0.350								
Heath Care	0.465	0.622	0.453	0.449							
Industrials	0.500	0.665	0.512	0.459	0.566						
Information Technology	0.504	0.612	0.888	0.357	0.460	0.531					
Materials	0.460	0.398	0.399	0.314	0.389	0.349	0.405				
Property Investment Trusts	0.534	0.693	0.564	0.450	0.573	0.558	0.572	0.474			
Telecommunication Services	0.602	0.742	0.591	0.457	0.606	0.705	0.604	0.440	0.639		
Utilities	0.461	0.591	0.809	0.345	0.480	0.512	0.747	0.377	0.547	0.607	
Market Index	0.443	0.577	0.483	0.435	0.444	0.479	0.496	0.324	0.501	0.559	0.469

Table 5.9 presents a time-series analysis for the 11 sectors. It is found that the industrials, information technology, materials, consumer discretionary and property sectors are significantly affected by systematic skewness. The following characteristics of these sectors may induce the asymmetry of their returns. Firstly, as the industrials, information technology and materials commonly have more volatile cash flows and are generally regarded as being cyclical, having high earnings leverage to levels of capital investment and operational expenditure, these sectors are very susceptible to market conditions and therefore to skewness risk. Secondly, the consumer discretionary and property investment trusts cater for individuals after their fiscal essentials for living have been met and therefore depend on extraneous spending and economic cycles. Such firms are likely to suffer most during an economic downturn.

The effects of the systematic kurtosis on sector returns are reported in column 3 of table 5.9. If the asset valuation of firms relies heavily on the present value of future growth opportunities, i.e. growth firms, then they are more susceptible to external shocks, which are the main driving force of volatility clustering and the emergence of fat tails (Campbell and Hentschel 1992). Consequently, the study finds that the systematic kurtosis effect is critical for the financials excluding property trusts, consumer discretionary, industrials, property investment trusts and telecommunication services. The significance of the kurtosis effect on the utilities sector comes as a surprise since the sector tends to be characterised as a mature sector with limited growth and therefore is less vulnerable to economic shocks. On the other hand, consistent with the notion that skewness and kurtosis have little effect on stable and matured industries, it is found that the energy and consumer staples sectors are the least sensitive to systematic skewness and systematic kurtosis. The overall evidence suggests that systematic skewness and systematic kurtosis are useful in explaining asset returns at both aggregate and sector levels.

Table 5.9 Impacts of systematic skewness and kurtosis on sector index returns for the period of January 1992 to May 2009

The table presents regressions results with Newey-West standard errors for the four-moment model: $R_t - r_{ft} = \alpha + \beta_1(R_{mt} - r_{ft}) + \beta_2 S_t + \beta_3 K_t$ where R_t is the index return of sector at time t, r_{ft} is the risk-free rate of return at time t, R_{mt} is the return of the market index at time t, S_t and K_t are the return premium for the systematic skewness and kurtosis factors respectively. Student's t-statistics are reported in the parentheses below the coefficient estimates. * and ** denote the statistical significance at 5 and 1 percent levels.

Sector	Market Premium	Systematic Skewness Premium	Systematic Kurtosis Premium	Adj. R^2
Consumer Discretionary	0.5784 (14.06)**	-0.1203 (-2.82)**	-0.0892 (-2.24)*	0.201
Consumer Staples	0.5868 (22.02)**	-0.0023 (-0.10)	-0.0108 (-1.31)	0.354
Energy	0.5368 (14.72)**	-0.0610 (-1.61)	-0.0183 (-0.52)	0.241
Financials-exc. Property Trusts	0.6386 (20.07)**	-0.0458 (1.39)	-0.1000 (-3.25)**	0.213
Heath Care	0.5968 (17.62)**	0.0185 (0.52)	-0.0824 (-2.51)**	0.211
Industrials	0.6071 (23.12)**	0.4062 (2.22)*	-0.1106 (-4.21)**	0.258
Information Technology	0.4774 (8.94)**	-0.1280 (-2.32)*	0.0077 (0.15)	0.256
Materials	0.5823 (15.76)**	-0.1401 (-3.67)**	-0.0554 (-1.55)	0.113
Property Investment Trusts	0.5545 (17.39)**	-0.0709 (-2.15)*	-0.1751 (-5.68)**	0.260
Telecommunication Services	0.6429 (15.50)**	-0.0072 (-0.17)	-0.1668 (-4.16)**	0.319
Utilities	0.5507 (15.30)**	-0.0561 (-1.50)	-0.1394 (-4.01)**	0.220

5.4 Conclusions

The study constructs a four-moment model to test the explanatory power of systematic skewness and systematic kurtosis in explaining patterns of asset returns. An examination of the pricing error of the four-moment model using the generalised multivariate approach proposed by Gibbons, Ross and Shanken (1989) shows that systematic skewness and systematic kurtosis can explain the pricing error of the CAPM. Furthermore, the test indicates that the four-moment model is better in explaining patterns of asset returns than the CAPM. Most importantly, the study in this chapter suggests that asset returns are generally mean-variance-skewness-kurtosis efficient.

The study examines the roles of systematic skewness and systematic kurtosis in explaining variation of time-series returns at both aggregate and industry levels. The overall results show that systematic skewness and systematic kurtosis are useful in explaining the variation of asset returns at both aggregate and industry levels. However, the degree of significance depends on stocks characteristics and market conditions. In particular, the roles of systematic skewness and systematic kurtosis in explaining the variation of expected returns are more vital in the downside market than in the upside market. Cyclical stocks, which are more susceptible to the market conditions, are more likely to be exposed to skewness risk while growth stocks are more vulnerable to fat-tail risk.

CHAPTER 6. ARE SYSTEMATIC SKEWNESS AND SYSTEMATIC KURTOSIS PRICING FACTORS? A CROSS-SECTIONAL ANALYSIS

6.1 Introduction

Given the results from Chapter 5 that asset returns are generally mean-variance-skewness-kurtosis efficient, do systematic skewness and systematic kurtosis also command significant risk premiums? To answer this question, the study examines the roles of these factors in explaining the variation of asset returns in cross-section.

To investigate the importance of systematic skewness and systematic kurtosis as pricing factors of asset returns in cross-section, this study adopts the Fama and MacBeth (1973) methodology. The Fama and MacBeth (1973) two-pass procedure is the first to interpret the CAPM as implying a basic linear relationship between stocks returns and the market beta, which should explain the cross-section of returns at a specific point of time. The advantages of this method are that it avoids the problem of spurious cross-sectional relations arising from statistical correlations between returns and the estimated betas and it maintains independence between the explanatory variables and the regression error term in the regression model (Shanken 1992).

To test the effectiveness of the four-moment model in justifying the variability of returns in cross-section, the Fama and MacBeth (1973) two-pass procedure is implemented in this study as follows. In the first pass, beta estimates are obtained from time-series regressions for each underlying asset and in the second pass, gammas are estimated cross-sectionally by regressing asset returns on the estimated betas. The averages of these gammas are interpreted as prices of risk factors. If the market, systematic skewness and systematic kurtosis are pricing factors, their gamma averages should be significantly different from zero.

Using the Fama and MacBeth procedure, this study finds that both systematic skewness and kurtosis systematic factors do command significant risk premium. Interestingly, it is found that when these factors are added to the CAPM model, they appear to be the dominant explanatory variables and make the market factor insignificant. This result is consistent with the findings of Brooks and Galagedera (2007), who argue that when the downside gamma, which is similar to the systematic skewness factor constructed in this study but only measures the downside of the return distribution, is included in the two-moment pricing model, the downside gamma is the only dominant explanatory variable. The analysis of sub-periods further reveals that the roles of these factors are particularly prominent when the Australian market experiences downturns.

The two-step approach devised by Fama and MacBeth (1973) has become a standard methodology in the finance literature for examining linear asset pricing relations. Despite its fundamental roles in modern asset-pricing empirical work, many researchers (Litzenberger and Ramaswamy 1979; Gibbons 1982; Shanken 1992; Kim 1995; Jaganathan and Wang 1996; Kan and Zhang 1997) have raised serious concerns about the errors-in-variables (EIV) problem in the second pass estimation. Criticisms focus on the unobservable market risk factor due to the fact that the market beta is estimated with errors in the first pass of the procedure, which then introduces EIV in the second pass. Shanken (1992) poses concerns about the asymptotic statistical properties of the finite sample distribution in the two-pass procedure. He argues that the EIV problems in the second pass estimators are severe in small samples. Dagenais and Dagenais (1997) argue that such EIV lead to inconsistency in OLS estimators, larger mean-squared errors and, most importantly, larger than intended sizes of Type I errors of Student's t-tests. Since the systematic skewness and systematic kurtosis measures are constructed as analogs of the market beta, they are likely to encounter the same problem as the market beta. Consequently, the precision of their gamma estimates in the second pass may be overstated.

Despite the importance of the EIV problem, there has been little attempt to correct the problem. Kim (1995) argues that the explanatory power of the book-to-market equity ratio for average stock returns reported by Fama and French (1992) is exaggerated under the traditional least squares estimation procedure, since the EIV problem results in an underestimation of the market risk and an overestimation of the other explanatory variables observed without errors (such as the size and the book-to-market factors). The same criticism applies to more general models such as Fama and French (1993) and Cahart (1997).

In this study, two approaches, the Shanken (1992) and the Dagenais and Dagenais (1997), are proposed to correct the EIV problem. The Shanken approach proposes that measurement errors in the beta estimates decline when the sample size increases and the second pass estimators would converge to the true values of gammas when the sample size approaches infinity. Therefore, Shanken attempts to derive a true asymptotic covariance of gammas for a finite sample which subsequently permits the validity of assessing the significance of pricing factors using OLS t-statistics. Using the Shanken approach to minimise the measurement errors in beta estimates, the study finds that the Fama and MacBeth procedure fails to reflect the measurement errors in the betas and therefore overstates the precision of the gamma estimates. This is consistent with the general findings of Shanken (1992), Kim (1995) and Shanken and Zhou (2007) that the EIV problem is critical and neglecting it may lead to false inferences when simple t-statistics are calculated to empirically validate or disprove a hypothesis based on the estimated parameters. Overall, although the measurement errors overstate the importance of systematic skewness and systematic kurtosis, these factors still retain their significance as pricing factors for asset returns.

Unlike Shanken (1992), Dagenais and Dagenais (1997) argue that the EIV problem may lead to the non-convergence of OLS estimators even when the sample size approaches infinity. Since the second pass estimators will not converge to the true value of gammas, it is impossible to generate the true asymptotic covariance of gammas. Dagenais and Dagenais (1997) suggest that an instrumental variable (IV) approach can solve the problem. However, it is not easy to identify available instrumental variables that are correlated with the true variables but unrelated to the measurement errors in this case. Reisersol (1950), Madansky (1959) and Bickel and Ritov (1987) propose that the easiest way to construct instrumental variables is to base them on information contained in higher order moments of data. Cragg (1997) and Dagenais and Dagenais (DD) (1997) propose that if regressors in the multivariate models exhibit skewness and/or kurtosis in their distributions, then the estimators based on moments order higher than two, called the Dagenais and Dagenais higher-moment estimators (DDHME), could help alleviate the EIV problem in the models. Since financial variables are often found to exhibit non-normality, this method offers an alternative approach to consider the issue of the EIV when the four-moment model is used to examine the variation of asset returns in cross-section.

When using DDHME to correct the EIV problem in the context of the four-moment model, the study finds that the significance of the market, systematic skewness and systematic kurtosis factors measured by traditional Fama and MacBeth cross-sectional regressions (CSR) is overstated. While the systematic skewness and systematic kurtosis premiums measured in the traditional two-pass CSR are significant in some sub-periods, the results do not hold for all these sub-periods when EIV problems are corrected using DDHME. Nevertheless, although the EIV correction leads to a diminished role of the market beta, systematic skewness and systematic kurtosis still retain their significance in explaining patterns of cross-sectional asset returns for the period from 1992 to 2009.

The remainder of this chapter is as follows. First, to examine the roles of systematic skewness and systematic kurtosis in explaining patterns of cross-sectional stock returns, the Fama and MacBeth (1973) two-pass procedure is employed for the four-moment model. As the EIV problem arises from the second pass of the estimation, two alternative solutions are proposed: the Shanken (1992) approach and the Dagenais and Dagenais (1997) higher-moment estimators approach. The empirical results and the discussion of these methods follow. Concluding remarks finish the chapter.

6.2 Methodology

In this section, the study uses the Fama and MacBeth (1973) two-pass regression approach to test the hypothesis of whether systematic skewness and systematic kurtosis are important pricing factors. As the EIV problem arises from the beta estimation in the second pass, the Shanken (1992) and the Dagenais and Dagenais (1997) methods are proposed to overcome the problem. Details of these tests are as follows.

6.2.1 The Fama and MacBeth Two-Pass Regressions for Cross-sectional Returns

In this section, the hypothesis that systematic skewness and systematic kurtosis are priced is tested. The regression analysis is in the spirit of Fama and MacBeth (1973) with a two-pass regression procedure. In the first pass of the procedure, portfolios are formed and risk factors are estimated. The risk estimates are then updated weekly. In the second pass, the study investigates whether these weekly risk estimates are on average significant. All of the tests in this procedure are predictive in the sense that the risk estimates are computed from the data for a period prior to the period of the portfolio returns on which the regressions are run.

In the first phase, to estimate the sensitivity of asset returns to the premium related to the market, the systematic skewness risk and the systematic kurtosis risk, rolling time-series regressions using 60 weekly observations are run:

$$R_{it} - R_{ft} = \alpha + \beta_{1,i}(R_{mt} - R_{ft}) + \beta_{2,i}S_t + \beta_{3,i}K_t + \varepsilon_{it}\,, \qquad (6.1)$$

where R_{it}, R_{ft} and R_{mt} are the return of portfolio i, the risk free-rate and the market return at time t respectively, S_t and K_t are the return premium of the systematic skewness and systematic kurtosis factors respectively and ε_{it} is the error term at time t.

The regressions generate risk estimates, β_i, for the next period based on the previous 60 weekly observations. After the first 60 observations, the components β_i are themselves updated weekly throughout the sample period. That is, they are recomputed each week throughout the examined period. Overall, with N weekly observations of the sample period, N-60 time-series observations of β_1, β_2 and β_3 are obtained.

In the second phase, to examine whether week-by-week risk sensitivity is a significant factor of the portfolio returns on average, the estimated values of β_i obtained from equation (6.1) are used as regressors to run the following cross-sectional regression for each week:

$$R_i - r_f = \alpha + \gamma_1\beta_{1,i} + \gamma_2\beta_{2,i} + \gamma_3\beta_{3,i} + \varepsilon_i \,, \qquad (6.2)$$

where R_i and r_f are the return of portfolio i and risk free rate of return respectively; $\beta_{1,i}$, $\beta_{2,i}$ and $\beta_{3,i}$ are the beta estimates of portfolio i generated from equation (6.1) and ε_i is the error term.

There are a totals of (N-60) cross-sectional regressions and (N-60) results for coefficient estimates, γ_i. A sample of mean estimate $\overline{\gamma_\imath}$ is calculated by averaging the coefficient estimates. If differences in expected returns can be explained by the betas β_i, the average coefficient estimates $\overline{\gamma_\imath}$ should be significantly different from zero. In other the words, hypotheses of zero gammas are tested:

$$H_o: \gamma_i = 0$$

If the market, the systematic skewness and the systematic kurtosis are considered as pricing factors, the null hypotheses of zero gammas should be rejected.

6.2.2 EIV Problems in Cross-Section of Expected Stock Returns

The two-pass procedure proposed by Fama and MacBeth (1973) has been widely used as a standard test for risk estimation in cross-section. The second pass is estimating the risk-return relation at a specific time t is using the model:

$$R_i - r_f = \alpha + \gamma_1\beta_{1,i} + \gamma_2\beta_{2,i} + \gamma_3\beta_{3,i} + \varepsilon_i, \qquad \forall\, i = 1,2..,N; \quad (6.3)$$

where β_1, β_2 and β_3 are the market beta, the systematic skewness and systematic kurtosis betas of underlying asset i, which are estimated by the first-pass regression procedure of the Fama and MacBeth (1973) in equation (6.1). $R_i - r_f$ is the excess return of underlying asset i and N is the total number of underlying assets.

Because of the unobservable estimated β_t, $\widehat{\beta}_{t-1}$ is used as a proxy for the unknown β_t in the second pass of the estimation. The independent variable β_t is measured with an error:

$$\widehat{\beta}_{t-1} = \beta_t + \zeta_{t-1}, \qquad (6.4)$$

where $\widehat{\beta}_{t-1}$ is the beta, which is either the market beta, the systematic skewness beta or the systematic kurtosis beta, estimated from the first-pass regression procedure using T (=60) time-series data available up to t-1 and ζ_{t-1} is a measurement error.

Although Shanken (1992) argues that the use of the predictive beta, $\widehat{\beta}_{t-1}$, in the CSR avoids the problem of spurious cross-sectional relations arising from statistical correlation between returns and the estimated betas and maintains the independence between the explanatory variables $\widehat{\beta}_{t-1}$ and the regression error term ε_t in the CSR model, the EIV problem is critical and may invalidate the significance of the explanatory power of the model.

To correct for the EIV problem, the two different approaches of Shanken (1992) and Dagenais and Dagenais (1997) are presented in the following sections.

Shanken (1992) assumes the regression regressors to be i.i.d and asymptotically normally distributed. He poses concerns about the asymptotic statistical properties of the finite sample distribution in the second pass procedure. He proposes that measurement errors in the beta estimates decline when the sample size increases and that the second pass estimators would converge to the true values of gammas when the sample size approaches infinity. Therefore, he attempts to derive a true asymptotic covariance of gammas which would subsequently permit the validity of assessing the significance of pricing factors using OLS t-statistics.

On the other hand, Dagenais and Dagenais (1997) argue that the EIV problem may lead to the non-convergence of ordinary least squares estimators even when the sample size approaches infinity. Since the second pass estimators cannot converge to the true value of gammas, it is impossible to generate the true asymptotic covariance of gammas. The Dagenais and Dagenais (1997) method relaxes Shanken's assumptions and considers the case of non-Gaussian distributions of the regressors. The goal of this approach is to construct error estimates representing the differences between the true values of the betas and the estimated betas. These error estimates are then included in the second pass regressions to permit the achievement of true values of gammas and therefore to correct inferences for the t-statistics.

6.2.2.1 EIV correction using Shanken (1992)

Shanken (1992) proposes that the Fama and MacBeth two-pass procedure for computing standard errors fails to reflect the measurement error in the market beta and overstates the precision of the market gamma estimate. Since systematic skewness and systematic kurtosis measures are constructed as analogs of the market beta, they are likely to encounter the same problem as the market beta. Consequently, the precision of their gamma estimates in the second pass is overstated. In the spirit of Shanken (1992), this section presents a method correcting for the EIV problem in the beta estimates in the second pass of the Fama and MacBeth (1973) regressions. In this method, the following assumptions are imposed:

(1) The regression regressors are i.i.d and asymptotically normally distributed.

(2) The disturbances of the cross-sectional regressions are independent over time and jointly distributed in each period with zero mean and a nonsingular residual covariance matrix.

The two-pass procedure in the general form is considered:

First pass: $R_t = \alpha + \sum_{j=1}^{K} \beta_{jt} f_{jt} + e_t$; (6.5)

Second pass: $R_i = \gamma_{0i} + \sum_{j=1}^{K} \gamma_j \beta_{ji} + \varepsilon_i \quad i = 1,..N$; (6.6)

In the first pass of the procedure, estimates of the betas are obtained by applying rolling time-series OLS regressions to equation (6.5). Let $\hat{\beta} = (\hat{\beta}_1,..,\hat{\beta}_K)$ be the N×K matrix of OLS slope estimates. For each period t, a cross-sectional regression of $R_i = (R_1,..,R_N)$ on $\hat{X} = [1_N, \hat{\beta}]$ is run to get estimates of γ_j. There are (T- 60) cross-sectional regressions in the second pass where T is the total number of time-series observations of R_i. The OLS t-statistics used for assessing the significance of pricing factors are computed as:

$$\hat{t}_j = \frac{\bar{\gamma}_j}{\hat{s}_j / \sqrt{T-60}} \quad j = 1,..,K; \tag{6.7}$$

where $\hat{s}_j$ is the sample standard deviation of γ_j.

Shanken (1992) argues that the two-pass procedure ignores estimation errors in the betas which create serious bias in small samples. Even if the measurement errors in beta decline as the sample increases, this bias does not disappear unless the sample size converges to infinity. Therefore, what matters is the rate of convergence, which is reflected by the asymptotic covariance of the second pass estimators. Shanken (1992) argues that by increasing the rate of convergence, the measurement of errors in the betas is minimised. He derives the adjusted asymptotic covariance-variance matrix of the second pass estimators for finite sample as follows:

Let $\Gamma = (\gamma_{1t}, .., \gamma_{kt})$ be the K×(T-60) matrix of γ_j.

The adjusted covariance-variance matrix is defined as:

$$\text{adj.var}(\Gamma) = V[1 + \bar{\Gamma}'(\Sigma_f)^{-1}\bar{\Gamma}] + \Sigma_f, \tag{6.8}$$

where V is the K×K covariance matrix of demeaned Fama-MacBeth coefficient estimates γ_i;

$\bar{\Gamma}$ is the vector of sample means of $\Gamma = (\gamma_{1t}, .., \gamma_{kt})$;

and Σ_f is the K×K covariance matrix of risk factor f.

The adjusted covariance matrix can be used to recalculate the t-statistics and therefore permits correct inferences from the t-statistics.

6.2.2.2 Dagenais and Dagenais (1997) Estimators for Asset Pricing Models with EIV

Unlike the Shanken (1992) method which assumes that the regressors in the linear models are asymptotically normally distributed, the Dagenais and Dagenais (1997) method of moments relaxes the i.i.d and normality assumptions of the Shanken (1992) method. This method considers cases of both Gaussian and non-Gaussian distributions of the regressors in order to correct the EIV problem. The method only assumes the normality condition for measurement errors.

Consider a multivariate model in a general matrix form:

$$Y = \alpha_N + \tilde{X}\beta + u, \tag{6.9}$$

where $\tilde{X}$ is the N×K matrix of explanatory variables measured without errors; N is the number of time-series observations and K is the number of regressors in the model; Y is the N×1 vector of observations of the dependent variable; u is the N×1 vector of normal residual errors where $u \sim N(0, \Sigma)$ and u is independent from the variables contained in $\tilde{X}$; α_N is the N×1 vector of intercepts. If matrix X is observed instead of $\tilde{X}$ where:

$$X = \tilde{X} + W \tag{6.10}$$

Equation (6.9) can be rewritten as:

$$Y = \alpha_N + X\beta + \eta \tag{6.11}$$

where $\eta = u - W\beta$ and W is the N×K matrix of errors in the variables.

It is assumed that W is uncorrelated with u but it is allowed that X may be correlated with u. Dagneais and Dagenais (1997) argue that this correlation and the EIV problem can lead not only to biased and inconsistent OLS estimators but also to an increase in sizes of Type I errors. Many studies have suggested using instrumental variables to obtain consistent estimators when information on the variance of these errors is not available. A common approach of the instrumental variables method is to acquire additional variables to serve as instruments for mis-measured regressors. However, in many situations no such variables are available. On the other hand, consistent estimators based on the original, unaugmented set of observables are usually available. Therefore, if the regressors in the multivariate models exhibit skewness and/or kurtosis in their distributions, Cragg (1997) and Dagenais and Dagenais (1997) argue that the estimators based on moments order higher than two could help construct consistent estimators and therefore alleviate the EIV problems in multivariate models. In this section, the method of Dagenais and Dagenais (1997) is used to construct linear estimators based on the third and fourth moments for the multivariate asset pricing models.

The DDHM estimators (α and β) are derived from the following orthogonality conditions:

$$E\left(Z'\eta / \sqrt{N} \right) = 0 \quad \text{when } N \to \infty \tag{6.12}$$

$$\text{where } Z = (i_N, z_1, z_2, z_3, z_4) \tag{6.13}$$

$$z_1 = x * x \tag{6.14}$$

$$z_2 = x * y \tag{6.15}$$

$$z_3 = y * y \tag{6.16}$$

$$z_4 = x * x * x - 3x\left[E\left(x'x / N \right) * I_N\right] \tag{6.17}$$

where the symbol * denotes the Hadamard element-by-element matrix multiplication operator[4]; variables x and y correspond to X and Y expressed in mean deviation form; i_N is a N×1 vector of ones; and I_N is the N×N identity matrix. The DDHM estimator is, in fact, an instrumental variable estimator, with Z serving as a matrix of instrumental variables. Unlike the Shanken (1992) method, the Dagenais and Dagenais method provides consistent estimations when the regressors are correlated with the error terms.

Let $\widehat{w}$ be the N×K matrix that represents the difference between the observed X and the estimated $\widehat{X}$. $\widehat{w}$ is estimated as:

$$\widehat{w} = X - \widehat{X} = X - Z(Z'Z)^{-1}Z'X \qquad (6.18)$$

To correct the EIV problem in the OLS regression, $\widehat{w}$ is added to the multivariate model and the OLS regression is re-run:

$$Y = \alpha_N + X\beta + \widehat{w}\varphi + \varepsilon \qquad (6.19)$$

where φ is the N×1 vector of parameters and ε is the N×1 vector of regression errors. The second pass CRS model is revisited using higher-moment estimators to correct the EIV problem in betas as follows:

$$R_i - r_f = \alpha + \gamma_1\beta_{1,i} + \gamma_2\beta_{2,i} + \gamma_3\beta_{3,i} + \varphi_1\widehat{w}_{1,i} + \varphi_2\widehat{w}_{2,i} + \varphi_3\widehat{w}_{3,i} + \varepsilon_i$$

$$\text{where} \quad i = 1,2..,N \qquad (6.20)$$

[4] If $A \in R^{m\times n}$ and $B \in R^{m\times n}$ then $A * B \in R^{m\times n}$ where the elements of $A * B$ are given by $(A * B)_{ij} = A_{ij}B_{ij}$. Note that if x is a matrix and y is a vector, then $x * y = (x_1 * y, x_2 * y, .., x_K * y)$ where x_j is the j^{th} column of x.

6.3 Results and Discussion

This section presents the empirical results generated from the methodology outlined in section 6.2. First, the cross-sectional regressions using the Fama and MacBeth (1973) approach are presented to test whether systematic skewness and systematic kurtosis are pricing factors for asset returns. Second, as the two-pass procedure is criticised for the EIV problem, the Shanken (1992) method is presented to correct for this problem. Third, the study presents an alternative solution for the problem using the Dagenais and Dagenais (1997) method.

6.3.1 Are Systematic Skewness and Systematic Kurtosis Pricing Factors for Asset Pricing? A Cross-sectional Regression Analysis

As described in the methodology, the Fama and MacBeth approach (1973) is used to examine whether systematic skewness and systematic kurtosis, analogous to the CAPM market beta, contribute significantly to the return premium in cross-section. In the first pass, the explanatory variables include returns of the market portfolio and mimicking portfolios for the systematic skewness and systematic kurtosis factors. The dependent variables are the 25 time-series excess returns of 25 portfolios, formed on the basis of systematic skewness and systematic kurtosis. The rolling regressions in the first pass generate beta estimates for the next period based on the previous 60 observations. The 25 time-series of the market beta, the skewness beta and the kurtosis beta are obtained from the rolling regressions. For each time point, a cross-sectional regression is run. The explanatory variables for the cross-sectional regressions are the market beta, the skewness beta and the kurtosis beta estimates. The dependent variable for the cross sectional regression is the portfolios' excess returns. The pricing factors for the market, the systematic skewness and the kurtosis betas are factor loadings or slopes generated from the cross-sectional regressions. From these cross-sectional regressions, three time-series of factor loadings for the market, the systematic skewness and the systematic kurtosis betas are obtained. The averages of these factor loadings over time, $\bar{\gamma}_1$, $\bar{\gamma}_2$ and $\bar{\gamma}_3$, are "ex post" prices of risk. They are equal to the "ex ante" prices plus any unexpected factor outcomes. If the market, the systematic skewness and the systematic kurtosis factors are pricing factors, their prices should be significantly different from zero.

The average estimated values of the betas and their t-statistics of the zero slope hypotheses are reported in table 6.1. Over the entire period, the average prices of the systematic skewness risk and the systematic kurtosis risk, represented by $\bar{\gamma}_2$ and $\bar{\gamma}_3$, respectively, are positive and significant. As the gamma estimates of systematic skewness and systematic kurtosis are significantly different from zero, the hypothesis of zero prices for these risk factors is rejected. This strongly suggests that skewness and kurtosis factors do have a predictive power for expected returns and that there is a significant positive trade-off between the expected return and the skewness and kurtosis risk for the period of 1992–2009. On the other hand, it is observed that, on average, the market effect is not significant once the systematic skewness and kurtosis factors are incorporated into the model. This result is consistent with the findings of Brooks and Galagedera (2007), who argue that when downside gamma, which is similar to our systematic skewness factor loading but only measures the downside of the return distribution, is included in the two-moment pricing model, the downside gamma appears to be the dominant explanatory variable and the market factor becomes insignificant.

To investigate whether the results for the whole period of 1992–2009 are general or whether they depend on the period examined and the business cycle, a sub-period analysis is developed using four different periods: 1992–1996, 1997–2001, 2002–2006 and 2007–2009.

It is interesting to observe that the systematic skewness and systematic kurtosis factors are heavily priced in the periods of 1997–2001 and 2007–2009 when the Australian market experienced downturns due to the Asian financial crisis in 1997, the deflation of the dot-com bubble in 2000–2001 and the global financial crisis in 2007–2009. On the other hand, the return premiums for systematic skewness and systematic kurtosis are not significant in the periods of 1992–1996 and 2002–2006 when the economy experienced expansionary phases. In terms of a risk-reward relationship, the evidence confirms the findings of Fabozzi and Francis (1977), Kim and Zumwalt (1979), Estrada (2002), Post and van Vliet (2006) and Brooks and Galagedera (2007) that the downside risk is more important to the investor's decision. With Australian stocks, this study finds that the downside risk is captured using both systematic skewness and systematic kurtosis.

Table 6.1 Summary results for Fama – McBeth cross-sectional regressions for the period from January 1992 to May 2009

The table reports the average estimates of the Fama-MacBeth cross-sectional regressions of portfolio returns on the market premium, systematic skewness and systematic kurtosis factors. To obtain portfolio betas for the risk factors in each week, rolling time-series regressions are run using 60 weekly observations: $R_{it} - R_{ft} = \alpha + \beta_{1i}(R_{mt} - R_{ft}) + \beta_{2i}S_t + \beta_{3,i}K_t$ where R_{it}, R_{ft} and R_{mt} are the return of portfolio i, the risk free-rate and the market return at time t respectively, S_t and K_t are the systematic skewness and kurtosis factors. After the first 60 observations, the risk factors are updated weekly, i.e. they are recalculated every week throughout the examined period. The Fama-MacBeth cross-sectional regression is run at the end of each week to determine if the risk factors have any predictive power for portfolio returns. The estimated values of β_1, β_2 and β_3 obtained from the rolling time-series regressions are used as regressors to run the cross-sectional regressions for each week: $R_i - r_f = \alpha + \gamma_1\beta_{1i} + \gamma_2\beta_{2i} + \gamma_3\beta_{3i}$. Finally, the null hypothesis of $\gamma_i = 0$ is tested. Student's t-statistics of the hypothesis are presented in parentheses and below the average weekly estimates $\overline{\gamma_i}$. * and ** denote statistical significance at 5 and 1 percent levels.

Period	$\overline{\gamma_1}$	$\overline{\gamma_2}$	$\overline{\gamma_3}$
1992-1996 **(t-statistic)**	-0.0065 (-0.77)	0.0024 (0.27)	-0.0101 (-0.86)
1997-2001	-0.0117 (-1.01)	0.0510 (2.14)*	-0.0526 (-2.37)**
2002-2006	-0.0039 (0.62)	0.0268 (1.03)	0.0233 (1.09)
2007-2009	0.0177 (1.97)*	0.0401 (2.03)*	0.0164 (3.77)**
1992-2009	-0.0718 (-1.51)	0.1785 (2.80)**	0.2918 (3.41)**

6.3.2 Cross-section of Asset Returns with the Errors-in-Variables Correction

The previous section evaluated the relative importance of systematic skewness and systematic kurtosis in explaining the variation of stock returns in cross-section. The study adopts the Fama and MacBeth (1973) two-pass regression procedure. In the first pass, beta estimates are obtained from the time-series regression for each underlying asset and in the second pass, gammas are estimated cross-sectionally by regressing asset returns on the estimated betas. However, many studies (Litzenberger and Ramaswamy 1979; Gibbons 1982; Shanken 1992; Kim 1995; Jaganathan and Wang 1996; Kan and Zhang 1997) have raised serious concerns about the estimation errors, known as the EIV problem, in the second pass of the procedure. This problem may lead to incorrect inferences when simple t-statistics are calculated to testify a hypothesis based on the estimated parameters. Two approaches, Shanken (1992) and Dagenais and Dagenais (1997), are implemented to minimise the problem. The differences between these two approaches are the assumptions they are based on. The former is based on the assumption that the regressors are asymptotically normally distributed over time and the latter considers both Gaussian and non-Gaussian distributions.

6.3.2.1 The Shanken (1992) approach for EIV correction

Table 6.2 presents the significance of premium estimates of the market, the systematic skewness and the systematic kurtosis factors with EIV correction. After adjusting for the EIV using the Shanken (1992) method, the t-statistic for the market risk premium becomes insignificant in every period examined. This confirms the findings of Shanken (1992) and Kim (1995) that the Fama and MacBeth (1973) two-pass procedure fails to reflect the measurement errors in the market beta and therefore overstates the significance of the market premium. It is observed that the two-pass procedure also overstates the significance of both the systematic skewness premium and the systematic kurtosis premium in every period examined. This is evidenced by the decreasing size of the t-statistics of these two factors in every period when the EIV problem is corrected. As the systematic skewness and systematic kurtosis measures are constructed as analogs to the market beta, it can be easily understood that if the Fama and Macbeth procedure fails to take into account the measurement error in the market beta estimate and overstates the significance of the market premium, it is also unsuccessful in reflecting these errors in the systematic skewness and systematic kurtosis beta estimates and therefore overstates the significance of these factor premiums. Interestingly, while systematic skewness still retains its significance as a pricing factor for asset returns, the systematic kurtosis premium becomes insignificant in the period of 1997–2001 once the EIV problem is corrected. On the other hand, both factors remain significant after the EIV problem is corrected in the period of 2007–2009.

Overall, the Fama and MacBeth two-pass procedure overstates the importance of the market and the systematic skewness and systematic kurtosis factors. Although it is arguable that the results for sub-periods depend on the length of the periods and the total number of observations, the results generally still support the previous findings that systematic skewness and systematic kurtosis command significant risk premiums and appear to be the dominant explanatory variables while making the market factor insignificant when they are included in the asset pricing model.

Table 6.2 Comparison of four-moment model estimates corrected for EIV using Shanken (1992) method and four-moment model estimates using traditional CSR

The table reports the significance of average risk premium estimates of the market premium, the systematic skewness and the systematic kurtosis factors using (1) week-by-week OLS cross-sectional regression model after estimating the betas in rolling regressions using 60 weekly observations (T=60) and (2) the Shanken (1992) estimators for cross-sectional regression model with EIV. The Shanken (1992) method adjusts the covariance matrix as: $\mathrm{adj.var}(\Gamma) = V[1 + \bar{\Gamma}'(\Sigma_f)^{-1}\bar{\Gamma}] + \Sigma_f$ where V is the K×K covariance matrix of demeaned Fama-MacBeth coefficient estimates γ_i; $\bar{\Gamma}$ is the vector of sample mean for $\Gamma = (\gamma_{1t}, \ldots, \gamma_{kt})$;and Σ_f is the K×K covariance matrix of risk factors f and K is the number of regressors. The adjusted covariance matrix is used to recalculate t-statistics in the null hypothesis of $\gamma_i = 0$. Student's t-statistics of the hypothesis are presented in parentheses and below the average weekly estimates $\bar{\gamma_i}$. * and ** denote statistical significance at 5 and 1 percent levels.

	Two-pass CSR			Shanken (1992) EIV Correction		
Period	$\bar{\gamma_1}$	$\bar{\gamma_2}$	$\bar{\gamma_3}$	$\bar{\gamma_1}$	$\bar{\gamma_2}$	$\bar{\gamma_3}$
1992-1996 (t-statistic)	-0.0065 (-0.77)	0.0024 (0.27)	-0.0101 (-0.86)	-0.0065 (0.50)	0.0024 -0.12	-0.0101 (0.51)
1997-2001	-0.0117 (-1.01)	0.0510 (2.14)*	-0.0526 (-2.37)*	-0.0117 (-0.37)	0.0510 (1.95)*	-0.0526 (-1.50)
2002-2006	-0.0039 (0.62)	0.0268 (1.03)	0.0233 (1.09)	-0.0039 -0.27	0.0268 (0.03)	0.0233 (0.92)
2007-2009	0.0177 (1.97)*	0.0401 (2.03)*	0.0164 (3.77)**	0.0177 0.47	0.0401 (1.98)*	0.0164 (2.42)**
whole period	-0.0718 (-1.51)	0.1785 (2.80)**	0.2918 (3.41)**	-0.0718 (0.17)	0.1785 (2.06)*	0.2918 (1.98)*

Table 6.3 Comparison of four-moment model estimates corrected for EIV using DDHME and four-moment model estimates using traditional CSR

The table reports the significance of average risk premium estimates of the market premium, the systematic skewness and the systematic kurtosis premium using: (1) week-by-week OLS cross-sectional regression model after estimating the betas in rolling regressions using 60 weekly observations (T=60) and (2) DDHME for cross-sectional regression model with EIV. The OLS cross-sectional regression model is $R_i - r_f = \alpha + \gamma_1\beta_{1,i} + \gamma_2\beta_{2,i} + \gamma_3\beta_{3,i} + \varepsilon_i$ where $i = 1,2..,N$. The equation for cross-sectional regression model corrected for the EIV problem is: $R_i - r_f = \alpha + \gamma_1\beta_{1,i} + \gamma_2\beta_{2,i} + \gamma_3\beta_{3,i} + \varphi_1\widehat{w}_1 + \varphi_2\widehat{w}_2 + \varphi_3\widehat{w}_3 + \varepsilon_i$ where $i = 1,2..,N$; $\beta_{1,i}, \beta_{2,i}$ and $\beta_{3,i}$ are obtained from rolling time-series regressions in the first-pass of CSR to estimate risk factors for each week and $\widehat{w}_i$ is the difference between the observed β_i and the estimated $\widehat{\beta_i}$ and is calculated using equation (6.18). The null hypotheses of $\gamma_i = 0$ and $\varphi_i = 0$ are tested respectively. Student's t-statistics of the hypothesis are presented in parentheses and below the average weekly estimates $\overline{\gamma_i}$ and $\overline{\varphi}_i$. * and ** denote statistical significance at 5 and 1 percent levels.

	Two-pass CSR			Higher Moment Estimators					
Period	$\overline{\gamma_1}$	$\overline{\gamma_2}$	$\overline{\gamma_3}$	$\overline{\gamma_1}$	$\overline{\gamma_2}$	$\overline{\gamma_3}$	$\overline{\varphi_1}$	$\overline{\varphi_2}$	$\overline{\varphi_3}$
1992-1996 (t-statistic)	-0.0065 (-0.77)	0.0024 (0.27)	-0.0101 (-0.86)	0.0004 (0.54)	-0.0015 (-0.27)	0.0397 (1.68)	0.4290 (1.24)	0.0055 (0.06)	-0.0058 (-1.19)
1997-2001	-0.0117 (-1.01)	0.0510 (2.14)*	-0.0526 (-2.37)**	0.0134 (0.85)	0.0154 (1.18)	0.0168 (1.16)	0.3562 (0.49)	-0.0118 (-0.64)	-0.0013 (-0.61)
2002-2006	-0.0039 (0.62)	0.0268 (1.03)	0.0233 (1.09)	-0.0066 (-0.65)	0.0030 (0.20)	0.0031 (0.37)	0.2784 (0.44)	0.0101 (0.52)	0.0076 (0.43)
2007-2009	0.0177 (1.97)*	0.0401 (2.03)*	0.0164 (3.77)**	0.0024 (1.07)	0.0140 (0.42)	0.0027 (0.10)	0.0824 (1.03)	0.0138 (2.00)*	0.0147 (1.60)
whole period	-0.0718 (-1.51)	0.1785 (2.80)**	0.2918 (3.41)**	-0.0324 (-0.66)	0.0271 (2.24)*	0.0184 (2.23)*	0.1891 (0.34)	0.0335 (1.16)	0.0421 (1.03)

6.3.2.2 Dagenais and Dagenais (1997) Estimators Approach for EIV Correction

An alternative method to correct the EIV problem in the second pass of the Fama-MacBeth methodology is to use DDHME. This method uses orthogonality conditions to construct error estimates representing differences between the true values of betas and the estimated betas. These error estimates are then included in the second pass regressions to estimate the unbiased values of gammas and therefore to correct inferences of t-statistics. The results of the second pass regressions with EIV correction can be found in table 6.3.

Table 6.3 compares the significance of the average estimated values of the market beta, the systematic skewness and systematic kurtosis premiums generated by using methods of OLS estimators and DDHME. First, the results are consistent with the findings of Shanken (1992) and Kim (1995) that the significance of the market premium is overstated when the traditional CSR ignores the measurement error in the market beta. This is indicated by the decreasing size of the t-statistic from the two-pass CSR method to the DDHME method in almost every period examined. While the systematic skewness and systematic kurtosis measured by traditional two-pass CSR are significant in both 1997–2001 and 2007–2009, their t-statistics are substantially reduced to become insignificant when the measurement errors are included in the model by using the DDHME method. Importantly, the measurement error of the systematic skewness is found to be significant at the 5 percent level for the 2007–2009 period. The finding for this period, however, is not consistent with the finding generated by the Shanken (1992) method as presented in the previous section. There is a possible reason for this inconsistency. The Shanken (1992) approach is based on the assumptions that the regression regressors are i.i.d and asymptotically normally distributed while the DDHME method relaxes the Shanken approach's assumptions and considers the case of non-Gaussian distributions of the regressors. Overall, although the EIV problems may overstate the significance of the cross-sectional results in sub-periods, this study finds that systematic skewness and systematic kurtosis remain as important pricing factors for asset returns for the whole period of 1992–2009.

6.4 Conclusions

The effectiveness of the four-moment model is tested for explaining the cross-sectional variation of asset returns. It is found that the Fama and MacBeth two-pass procedure for estimating factor prices can be modified to accommodate additional risk measures such as systematic skewness and systematic kurtosis. Using this procedure, the study finds that systematic skewness and systematic kurtosis do command significant risk premia and therefore they are pricing factors for asset returns. Importantly, when systematic skewness and systematic kurtosis are included in the CAPM, these factors appear to be the dominant explanatory variables and make the market factor insignificant.

Despite the fundamental roles played by the two-pass methodology in modern asset pricing, the approach has been criticised for the imprecision of estimation of the parameters of the cross-sectional regressions and hence the validity of conclusions derived from these estimates. In particular, the beta estimates measured in the second pass of the estimation are subject to the EIV problem which may lead to inconsistency of OLS estimators and therefore may overstate the significance of the explanatory power of the four-moment model. Two alternative approaches are proposed to minimise this problem. The Shanken (1992) approach aims to derive the adjusted covariance matrix of regressors when the sample size approaches infinity while the Dagenais and Dagenais (1997) approach seeks to generate parameters proxying for the difference between the true value of the betas and their estimates. The results from these two approaches strongly suggest that the significance of the market, the systematic skewness and the systematic kurtosis measured by traditional Fama and MacBeth cross-sectional regressions is overstated. Nevertheless, systematic skewness and systematic kurtosis still retain their significance as pricing factors for asset returns in cross-section for the 1992–2009 period.

CHAPTER 7. CAN SYSTEMATIC SKEWNESS AND SYSTEMATIC KURTOSIS CAPTURE MARKET RISK ASYMMETRY?

7.1 Introduction

The instability of the time-series estimates of parameters of the CAPM has a literature dating from the 1970s. The studies of Klemkosky and Martin (1975), Levy (1977), Fabozzi and Francis (1977, 1979) and Bhardwaj and Brooks (1993) pose questions on the validity of the Sharpe-Litner CAPM. They argue that the asymmetric variation in the market risk factor due to premium changes between bull and bear markets is not explained by the Sharpe-Litner CAPM. More recently, the studies of Campbell *et al.* (2001), Bekaert and Harvey (2000) and Ang and Chen (2002, 2007) document the asymmetry between upside and downside market betas. Pettengill *et al.* (1995) propose that if there is no systematic relationship between asset returns and the market beta, continued reliance on the beta as a measure of risk is inappropriate.

While the importance of the market risk asymmetry is widely documented, are there any links between the market risk asymmetry and the roles of systematic skewness and systematic kurtosis factors in asset pricing as documented in the previous chapters? One clue that pushes this study in the direction of higher moments to explain risk asymmetry is that skewness measures the asymmetry of the return distribution while kurtosis measures the abnormal returns at the tails of the return distribution caused by investors reacting strongly to extreme market conditions. In other words, skewness and kurtosis are associated with non-normalities of the return distribution and therefore there is a possibility that downside and upside betas are strongly correlated with skewness and kurtosis. In fact, Harvey and Siddque (2002) provide some evidence that skewness can capture some asymmetry in risk. The goal of this study is to go one step further and examine whether both skewness and kurtosis together can effectively capture the asymmetry of market risk. The study includes both skewness and kurtosis because recent studies (McNeil and Frey 2000; Bali 2003; Cotter 2004) document that measures of market risk are largely influenced by extreme market returns and negative extreme returns occur more often than positive extreme returns.

Although the topic of varying beta risk was mentioned 30 years ago, most empirical studies focus on the U.S. market while little has been done to verify if conclusions drawn from the U.S. market are also valid for markets outside the U.S. Australian evidence of the varying beta risk can only be found in a few studies such as Faff, Lee and Fry (1992), Brooks, Faff and Lee (1992, 1994), Pop and Warrington (1997) and Brooks, Faff and McKenzie (1998). One common feature of these studies is that they only focus on conditional time-varying beta and therefore they do not provide evidence as to specifically when major changes in the market beta occur and how beta instability affects the risk premium. Most importantly, there are no studies in the literature investigating the link between beta instability and systematic skewness and systematic kurtosis. This is an interesting research area to which the study in this chapter seeks to contribute.

This study applies the dual-beta asset pricing model proposed by Bharadwai and Brooks (1993) in the context of weekly Australian stocks from 1992 to 2006 to examine beta instability. The dual-beta model is in fact the Sharpe-Litner CAPM, adding a binary variable to segment the stock performance into the upside and downside markets. The results firmly reject the hypothesis of beta stability and support the findings of Campbell *et al.* (2001), Bekaert and Harvey (2000) and Ang and Chen (2002, 2007), who document the asymmetry between upside and downside betas. The results are consistent with Pettengill, Sundaram and Mathur (1995), who argue there exists a significant and direct beta-return relationship in the upside market and a significant inverse relationship between the beta and the return in the downside market. Most importantly, the results show that the downside risk and the upside risk are priced asymmetrically and the return premium for the downside risk is significantly higher than the premium for the upside risk.

In an attempt to establish the link between market risk asymmetry and systematic skewness and systematic kurtosis, this study investigates whether the asymmetry in the risk premium documented in the two-moment framework still persists if the asset pricing model incorporates systematic skewness and systematic kurtosis. Interesting conclusions are drawn from this analysis. It is found that when these factors are added into the dual-beta asset pricing model, the asymmetry in the market risk factor becomes insignificant. This implies that systematic skewness and systematic kurtosis can capture beta asymmetry effectively and therefore they can proxy for the market risk asymmetry caused by the changes in market conditions. Since the use of the dual-beta asset pricing model proposed by Bhardwaj and Brooks (1993) requires the specification of bull and bear market conditions, the model is sensitive to the way bull and bear periods are constructed. The four-moment model has advantages over the dual-beta model because it provides a genuine method to capture the asymmetry in risk-factor loadings caused by changes in market conditions without having to assume any specific formation methods to construct bull and bear periods.

Finally, the method of creating portfolios on the basis of various risk characteristics creates a systematic approach to identify the relationship between the beta asymmetry and systematic skewness and systematic kurtosis. Kothari, Shanken and Sloan (1995) and Howton and Peterson (1998) argue that the significance of the market beta is sensitive to the way it is estimated. Being aware of this matter, this study estimates the market beta on the basis of the downside, upside and regular market risk characteristics. Dependent portfolios are constructed by sorting stocks by each of these betas. Based on the findings under different market conditions, the study suggests that the downside risk appears to be more important to investors than the upside risk, and therefore it makes more sense to use downside beta portfolios to examine the risk-return relationship.

Overall, the aim of this study is to answer the third research question proposed in Chapter 1 as to whether systematic skewness and systematic kurtosis can proxy for the asymmetry in risk factor loadings. The main contribution of this study is that this is the first study in the literature to successfully show that systematic skewness and systematic kurtosis can capture effectively the market risk asymmetry caused by changes in market condition between bull and bear markets. Therefore, the study proposes the use of the four-moment model to capture risk asymmetry in asset pricing. Finally, the study recommends that, because the downside volatility risk dominates the upside volatility risk, the risk-return relationship is better revealed when dependent portfolios are formed on the basis of downside risk characteristics.

The chapter is organized as follows. Section 2 presents the methodology to firstly construct beta measures based on different risk characteristics, secondly to test the significance of beta asymmetry using different asset pricing models, and thirdly to investigate relationships between systematic skewness, systematic kurtosis and the beta asymmetry. Section 3 presents empirical results and discussion. Section 4 concludes the chapter.

7.2 Methodology

Economists have long recognized that investors care differently about downside losses versus upside gains in absolute values. To examine how an investor's expected return varies across different market conditions, it is necessary to examine the relationship between the expected return and its covariance with the market returns. A convenient way to first look at this matter is to construct beta measures in bull and bear markets and examine whether these measures have any explanatory power for describing patterns of asset returns. A crucial step that may affect the result of this analysis is the construction of bull and bear periods. Fabozzi and Francis (1977) define a bull period in which the market return is non-negative and a bear period in which the market is positive. It is argued that this definition would bias the relationship between the expected return and the bull and bear market betas if the zero return happens to be at either tails of the market return distribution. Bawa and Lindenberg (1977) define a period in which the market return is greater than its mean as a bull period and define a bear period in which the market is less than its mean. This study argues that the periods when the market returns are around its mean are trendless and probably lead to an unclear relationship between the expected return and the betas measured in the bull and bear markets. Instead, the study constructs bull and bear periods using a quantile approach of the probability distribution. Based on periods of upside and downside markets, the upside and downside betas are estimated. The study also constructs systematic measures of skewness and kurtosis to investigate the relationships between these measures and the beta measures. Using portfolios constructed on the basis of the betas, systematic skewness and systematic kurtosis, the study examines whether systematic skewness and systematic kurtosis factors can proxy for the market risk asymmetry. Details are as follows.

7.2.1 Regular, Downside and Upside Beta Measures

This study constructs bull and bear periods by using a quantile method as follows. The examined period is partitioned into three sub-periods:

(1) periods when the market moves up substantially;

(2) periods when the market moves down substantially; and

(3) periods when the market moves neither up nor down substantially.

The lowest 25 percent of the market return distribution is viewed as periods when the market moves down substantially while the highest 25 percent of the market return distribution is viewed as periods when the market moves up substantially. The 50 percent in the middle of the market return distribution is viewed as periods when the market is essentially directionless, and therefore it is not used in the analysis of beta asymmetry. Prior to any regression analyses in subsequent sections, appropriate market betas for individual stocks are determined on the basis of various market risk characteristics as follows:

1. The conventional market beta is defined as:

$$\beta = \frac{\mathrm{Cov}(R_{it}, R_{mt})}{\mathrm{VAR}(R_{mt})} . \tag{7.1}$$

2. The downside beta is defined as:

$$\beta^{-} = \frac{\mathrm{Cov}(R_{it}, R_{mt} | R_{mt} \in Q_1)}{\mathrm{VAR}(R_{mt} | R_{mt} \in Q_1)}. \tag{7.2}$$

3. The upside beta is defined as:

$$\beta^{+} = \frac{Cov(R_{it}, R_{mt} | R_{mt} \in Q_4)}{VAR(R_{mt} | R_{mt} \in Q_4)}. \qquad (7.3)$$

where R_{it} and R_{mt} are the return of asset i and the market return at time t respectively; Q_1 and Q_4 are the lowest 25th percentile and the highest 25th percentile of the market return distribution respectively.

Recent studies by McNeil and Frey (2000), Bali (2003) and Cotter (2004) document that as skewness and kurtosis are associated with non-normalities in the return distribution, it is important to investigate the relationship between skewness and kurtosis and the regular, downside and upside market betas. Kraus and Litzenberger (1976) define the systematic measure of skewness as an analog to the market beta as follows:

4. $$S_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^2]}{\{R_m - E(R_m)\}^3}. \qquad (7.4)$$

Using the methodology suggested by Kraus and Litzenberger (1976), this study defines systematic measure of kurtosis as:

5. $$K_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^3]}{\{R_m - E(R_m)\}^4}. \qquad (7.5)$$

While the market beta is interpreted as a measure of sensitivity of asset returns to the market returns, the downside (upside) beta is interpreted as a measure of sensitivity of asset returns to the market returns when the market is in its downturn (upturn). The systematic skewness (kurtosis) is defined as a component of an asset's skewness (kurtosis) related to the market portfolio's skewness (kurtosis).

7.2.2 Data and Portfolio Construction

The study in this chapter uses the sample of all listed stocks on the ASX for the period of 1992–2006. The period of 2007–2009 is excluded in this chapter because the unusual conditions of the global financial crises may bias the findings and implications of the chapter. This is important as the purpose of this chapter is to investigate the existence of the market risk asymmetry caused by changes in up and down market conditions. The inclusion of very extreme negative outliers such as the 2007–2009 period would increase the possibility of finding significant risk asymmetry, which does not necessarily represent common characteristics of the risk factors in the period of 1992–2009. Weekly returns are collected for 2185 stocks from Datastream. The ASX300 index returns and 90-day Bank-Accepted bill rates are used as proxies for the market returns and the risk-free rate returns respectively.

Like many other asset pricing studies, to reduce information loss in testing the risk-return relationship, this study uses portfolios constructed on the basis of various market risk characteristics as underlying assets to examine the risk-return relationship. As Ang *et al.* (2006) emphasise that a relationship between factor sensitivities and asset returns should hold for an average stock (equal-weighting) and an average dollar (value-weighting), the study only uses with equal-weighted portfolios to unfold the risk-return relationship across different market conditions. The β, β^- and β^+ of each stock are computed and ranked in an ascending order. For each risk characteristic, the stocks are sorted into five quintiles with an equal number of stocks in each quintile. This creates three sets of portfolios: five regular beta portfolios, five downside beta portfolios and five upside beta portfolios. Quintile 1 comprises stocks with lowest risk measures (i.e. low beta/downside beta/upside beta measures) and quintile 5 comprises stocks with highest risk measures. If there is a relationship between risk and return, the patterns between the returns of these portfolios and their beta measures, which associate with exposure to their corresponding market risk, should be observed in the regression analysis. For example, the CAPM implies that stocks covarying strongly with the market have contemporaneously high average returns over the same period. This means that the CAPM predicts an increasing relationship between average returns and the value of the market beta. More generally, if a factor is to be included in an asset pricing model, the patterns between the average returns and its sensitivity to the risk which the factor represents should be observed.

7.2.3 Asymmetric Market Betas over Bull and Bear Markets

The suitability of using the beta coefficient as a proper measure of the systematic risk is dependent upon the assumption that the beta coefficient is stable over time. It is, however, questionable whether the CAPM model is robust when the market risk varies across bull and bear markets. This section attempts to show that the market risk is asymmetric in bull and bear periods and as a result, the market risk premium is asymmetric. The study uses the Bhardwaij and Brooks (1993) approach to test whether patterns of average returns are more likely to be captured by a varying risk market model that accounts for the asymmetric risk in bull and bear markets than to be captured by a constant risk model such as the CAPM. The study employs the CAPM and the dual-beta asset pricing market model defined by Bhardwaij and Brooks (1993) to test this hypothesis.

The CAPM with the constant market beta is specified as:

$$R_t - R_{ft} = \alpha + \beta(R_{mt} - R_{ft}) + e_t. \quad (7.6)$$

The Bhardwaij and Brooks (1993) dual-beta model is specified as:

$$R_t - R_{ft} = \alpha + \alpha' d_t + \beta(R_{mt} - R_{ft}) + \beta' d_t(R_{mt} - R_{ft}) + e_t, \quad (7.7)$$

where R_t, R_{ft} and R_{mt} are the asset return, the risk free-rate and the market return respectively; d_t is a binary variable which is equals to 1 in bull periods and zero in bear periods; and e_t is the error where $e_t \sim N(0, \sigma^2)$.

The intercept α and the slope β of the CAPM in equation (7.6) represent the intercept and the market beta respectively. The estimate α' in equation (7.7) represents the intercept difference between bull and bear markets. Likewise, the estimate β' represents the premium difference of the market risk between bull and bear markets. The statistical significance of β' determines whether downside risk and upside risk are priced asymmetrically.

7.2.4 Can Systematic Skewness and Systematic Kurtosis Proxy for Beta Asymmetry?

Skewness and kurtosis are largely associated with non-normalities in the return distribution (Bali 2003; Cotter 2004). This motivates the study to investigate to what extent downside and upside betas are correlated with the systematic skewness and systematic kurtosis; and whether these two factors can proxy for the market risk asymmetry caused by changes in the market conditions. In this section, the study constructs the four-moment model by adding systematic skewness and systematic kurtosis to the CAPM. The four-moment model and its dual-beta versions are then used to test the hypothesis of whether systematic skewness and systematic kurtosis can proxy for the beta asymmetry.

The four-moment model (Model 1) is specified as:

$$R_t - R_{ft} = \alpha + \beta_1(R_{mt} - R_{ft}) + \beta_2 S_t + \beta_3 K_t + e_t \qquad (7.8)$$

where R_t , R_{ft} and R_{mt} are the asset return, the risk free-rate and the market return at time t respectively, S_t and K_t are systematic skewness and systematic kurtosis premiums respectively.

To investigate the asymmetry in risk factor loadings of the four-moment model, dummy variables proxying premium differences between bull and bear markets of the risk factors are added to the model. This study examines the following versions of dual-beta four-moment models:

Model 2:

$$R_t - R_{ft} = \alpha + \alpha' d_t + \beta_1(R_{mt} - R_{ft}) + \beta'_1 d_t(R_{mt} - R_{ft}) + \beta_2 S_t + \beta_3 K_t + e_t \,, \qquad (7.9)$$

where α' represents the difference of the intercepts and β'_1 represents the difference of the market betas between bull and bear markets. This model is used to examine if the beta asymmetry in the two-moment framework (i.e. equation (7.7)) still persists when systematic skewness and systematic kurtosis are added to the CAPM.

Model 3:

$$R_t - R_{ft} = \alpha + \alpha' d_t + \beta_1(R_{mt} - R_{ft}) + \beta'_1 d_t(R_{mt} - R_{ft}) + \beta_2 S_t + \beta'_2 d_t S_t + \beta_3 K_t + \beta'_3 d_t K_t + e_t \,, \qquad (7.10)$$

where α' represents the difference of the intercepts and β'_1, β'_2 and β'_3 represent the differences of the market, the systematic skewness and the systematic kurtosis betas between bull and bear markets respectively. This model is used to examine two hypotheses:

(1) whether the beta asymmetry in two-moment framework still persists when systematic skewness and systematic kurtosis are added into the CAPM; and

(2) whether risk premiums for the systematic skewness and systematic kurtosis factors are asymmetric between bull and bear markets.

To test whether changes in market conditions have any significant effects on the risk factors of the four-moment model, F-tests of joint zero binary variable are employed for the following hypotheses:

H1: $\beta'_2 = \beta'_3 = 0$, against the alternative hypothesis H1a: $\beta'_2 \neq 0$ or $\beta'_3 \neq 0$.

H2: $\beta'_1 = \beta'_2 = \beta'_3 = 0$,against the alternative hypothesis H2a: $\beta'_1 \neq 0$ or $\beta'_2 \neq 0$ or $\beta'_3 \neq 0$.

The first hypothesis tests whether systematic skewness and systematic kurtosis risk factors are priced symmetrically between bull and bear markets. The second hypothesis tests whether all risk factors in the four-moment model, i.e. the market, the systematic skewness and the systematic kurtosis, are priced symmetrically between bull and bear markets. These hypotheses help determine whether these factors can proxy for risk asymmetry caused by changes in the market conditions.

7.3 Empirical Results and Discussion

This section presents the empirical results and discussion using the methodology outlined in the previous section. First, summary statistics of the dependent portfolios and their return relationships with the upside, downside and regular market betas are presented. Second, the correlations between the betas and systematic skewness and systematic kurtosis are investigated. Third, the asymmetric market risk is examined using a time-series analysis with the dual-beta asset pricing model. The analysis of whether systematic skewness and systematic kurtosis can capture effectively asymmetry in risk factor loading finishes the section.

7.3.1 Preliminary Results

Table 7.1 presents summary statistics of the weekly returns of the portfolios constructed on the basis of regular, downside and upside betas. The sample consists of 2185 stocks and each portfolio comprises 780 weekly observations. Low beta portfolios comprise stocks with low betas while high beta portfolios comprise stocks with high betas. In addition to the mean and standard deviation of the portfolio returns, total skewness and excess kurtosis are reported as well as the Jarque-Bera test for normality.

As can be seen from table 7.1, there is no obvious relationship between the portfolio returns and the market betas. However, it can be observed that there is a positive relationship between the return range and the market betas. Low beta portfolios have smaller return ranges while high beta portfolios have larger return ranges. Consistent with the patterns of the return ranges, high beta portfolios have higher standard deviations compared to those of low beta portfolios. That is, returns of high beta portfolios are more dispersed than those of low beta portfolios. Total skewness of portfolio returns is mostly negative while excess kurtosis is largely positive. This implies that the return distributions of the portfolios are leptokurtic and mostly negatively skewed. The rejection of the normality hypothesis using the Jarque-Bera test in every portfolio reinforces suggestions of several studies such as Harvey and Zhou (1993) and Richardson and Smith (1993), which propose that portfolio returns do not conform to a normal distribution.

Overall, the descriptive statistics suggest the need for further investigation into the relationship of volatility risk and asset returns in bull and bear markets. In the subsequent sections, the study investigates to explain why asset returns are more dispersed in the downside than in the upside of the market and whether this behaviour challenges the traditional ways of pricing assets. Finally, the descriptive results suggest that non-normality of portfolio returns is common, so the role of higher moments of returns, especially skewness and kurtosis, are important in explaining return patterns of these portfolios.

Table 7.1 Summary statistics of returns of portfolios formed on the basis of regular beta, upside beta and downside beta: January 1992 to December 2006

The sample consists of 2185 stocks listed in the 1992–2006 period. Each portfolio comprises 780 weekly observations and is constructed on the basis of regular, downside and upside betas. Low β (β^- or β^+) portfolio contains stocks with low β (β^- or β^+) while high β (β^- or β^+) portfolio contains stocks with high β (β^- or β^+). The Jarque-Bera normality test is a test of whether the stock returns are normally distributed.

Portfolio	Mean	Median	Maximum	Minimum	Return Range	Std. Dev.	Skewness	Excess Kurtosis	Jarque-Bera	Probability	Observations
Portfolios sorted by Regular Beta											
Low β	0.04%	0.02%	3.77%	-3.27%	7.05%	0.94%	-0.13	3.96	132	0.00	780
2	0.09%	0.17%	3.85%	-6.86%	10.71%	0.99%	-0.69	7.05	596	0.00	780
3	0.09%	0.13%	5.64%	-11.60%	17.25%	1.44%	-0.96	9.34	1431	0.00	780
4	-0.01%	0.12%	10.76%	-15.79%	26.54%	2.32%	-1.20	10.20	1879	0.00	780
High β	-0.08%	0.11%	19.20%	-29.08%	48.28%	3.65%	-1.41	14.01	4213	0.00	780
Portfolio sorted by Downside Beta											
Low β^-	-0.03%	0.04%	5.32%	-8.15%	13.47%	1.35%	-0.55	5.99	331	0.00	780
2	0.10%	0.16%	3.44%	-6.25%	9.69%	0.98%	-0.40	5.80	276	0.00	780
3	0.11%	0.20%	5.00%	-10.89%	15.88%	1.49%	-0.95	7.60	806	0.00	780
4	0.03%	0.18%	10.59%	-17.92%	28.51%	2.42%	-1.26	11.75	2700	0.00	780
High β^-	-0.12%	0.04%	18.31%	-31.12%	49.43%	3.38%	-1.92	18.88	8694	0.00	780
Portfolio sorted by Upside Beta											
Low β^+	-0.03%	0.04%	6.10%	-12.45%	18.55%	2.01%	-0.82	6.54	496	0.00	780
2	0.07%	0.18%	6.29%	-10.61%	16.91%	1.44%	-1.28	11.15	2374	0.00	780
3	0.14%	0.25%	4.43%	-9.52%	13.95%	1.23%	-1.34	9.46	1593	0.00	780
4	0.06%	0.13%	6.41%	-11.52%	17.93%	2.63%	-0.96	8.27	1026	0.00	780
High β^+	-0.14%	-0.10%	17.07%	-20.12%	37.19%	2.74%	-0.76	12.60	3078	0.00	780

7.3.2 Relationships between Returns and Regular, Downside and Upside Betas

In attempting to investigate the asymmetric risk-return relationship, the study first examines patterns between average returns and standard deviations of portfolio returns constructed on the basis of regular, downside and upside betas. The periods in which the market returns are in the lowest 25 percent of the market return distribution are defined as bull periods and the periods in which the market returns are in the highest 25 percent of the market return distribution are defined as bear periods. The portfolios are constructed as follows. First, the downside beta of each stock is calculated based on its bear periods and the upside beta is calculated based on its bull periods. The regular beta is calculated based on the whole period from 1992 to 2006. Second, individual stocks are sorted into five quintile portfolios on the basis of their regular betas, downside betas and upside betas. Each quintile portfolio consists of 437 stocks. Third, average returns and standard deviations of these portfolios are computed under different market conditions. The spreads of the average returns and the standard deviations between the highest and low beta portfolios sorted by three different risk characteristics are reported in table 7.2

Table 7.2 presents preliminary results for patterns of stock returns and standard deviations when stocks are sorted on the basis of regular, downside and upside market betas. It is observed that average returns are positive in every portfolio in the bull periods while they are negative in the bear periods. For portfolios sorted on the basis of the regular beta and the downside beta, their standard deviations increase almost monotonically from the lowest beta to the highest beta portfolios respectively. It is observed that although stocks are sorted on the basis of different market risk characteristics, low beta (i.e. low β, β^- or β^+) portfolios outperform high beta portfolios in the bear periods while underperforming them in the bull periods. Average returns generally decrease from the low to the high beta portfolios in the bear periods while this trend is reversed in the bull periods. In other the words, high beta portfolios have a potential to collect a big win in the upside market but, on the other hand, they are prepared to bear a huge loss in the downside market.

To examine if the risk-return relationship can be observed by using portfolios constructed on various risk characteristics, table 7.2 presents the hypothesis test of return equality between the highest and lowest beta portfolios using t-tests. Overall, the hypothesis is rejected for bull and bear periods but not for the whole period. This is because the average return measured for the whole period is considered as the average of the bull and the bear returns. As the risk-return relationships in the bull and the bear markets are opposite to each other, they partially cancel each other out in the average calculation, causing the corresponding risk-return relationship to be insignificant for the whole period. On the other hand, the significance of the t-tests for the bull and the bear periods implies that analysing the risk-return relationship by segmenting the market according to its conditions is far better than the traditional method of examining the entire period. This implies that the relationship between the market beta and asset returns is better reflected if the market is segmented into bull and bear periods because low beta portfolios significantly outperform high beta portfolios in the bear periods while underperforming them in the bull periods.

Table 7.2 Relationships between portfolio returns and regular, upside and downside betas for the period of January 1992 to December 2006

The table presents the equal-weighed returns and risk characteristics of stocks sorted by the β, β^- and β^+. For each risk characteristic, stocks are sorted into five quintiles with an equal number of stocks in each quintile. The columns labeled "Average Return" and "SD" report the average weekly return and the standard deviation of each quintile respectively. P-value reports the probability of the t-tests for equality.

		Whole period		**Bear Period (190 weeks)**		**Bull Period (190 weeks)**	
	No. of stocks	Average Return	SD	Average Return	SD	Average Return	SD
		Portfolios sorted by Regular Beta					
Low β	437	0.04%	0.94%	-0. 04%	1.01%	0.11%	0.88%
2	437	0.09%	0.99%	-0.48%	1.08%	0.53%	0.86%
3	437	0.09%	1.44%	-1.00%	1.56%	0.93%	1.15%
4	437	-0.02%	2.32%	-1.67%	2.68%	1.30%	1.70%
High β	437	-0.08%	3.65%	-2.57%	4.28%	1.94%	2.86%
High-Low		**-0.12%**	**2.71%**	**-2.53%**	**3.27%**	**1.79%**	**1.98%**
P-value of t-test		*0.356*	*0.000***	*0.000***	*0.000***	*0.000***	*0.000%*
		Portfolios sorted by Downside Beta					
Low β^-	437	-0.03%	1.35%	-0.76%	1.54%	0.58%	1.14%
2	437	0.10%	0.98%	-0.61%	0.95%	0.71%	0.83%
3	437	0.11%	1.50%	-1.00%	1.57%	0.98%	1.07%
4	437	0.03%	2.41%	-1.51%	2.85%	1.13%	1.85%
High β^-	437	-0.12%	3.37%	-2.01%	4.45%	1.17%	2.65%
High-Low		**-0.09%**	**2.02%**	**-1.25%**	**2.91%**	**0.59%**	**1.51%**
P-value of t-test		*0.517*	*0.000***	*0.000***	*0.000***	*0.005***	*0.000***
		Portfolios sorted by Upside Beta					
Low β^+	437	-0.03%	2.01%	-1.09%	2.40%	0.57%	1.74%
2	437	0.07%	1.43%	-0.90%	1.69%	0.77%	1.15%
3	437	0.14%	1.23%	-0.78%	1.42%	0.86%	0.85%
4	437	0.06%	1.63%	-1.10%	1.79%	0.98%	1.30%
High β^+	437	-0.14%	2.74%	-1.80%	2.99%	1.19%	2.63%
High-Low		**-0.09%**	**0.63%**	**-0.75%**	**0.59%**	**0.62%**	**0.89%**
P-value of t-test		*0.370*	*0.023**	*0.011**	*0.003***	*0.007***	*0.009%*

Table 7.2 provides evidence that standard deviations and high-low return spreads are substantially higher in absolute values when they are measured in the bear market compared to the same measures in the bull market. This suggests that stock returns react asymmetrically to changes of the market conditions. Therefore it is inaccurate to use the regular beta to examine the risk-return relationship as the regular beta implies “symmetry” of the stock reaction in both directions of the market while the investor’s reaction to the market is asymmetric. When underlying portfolios are formed on the basis of various risk characteristics, the table shows that the risk-return relationship is better reflected when portfolios are formed on the basis of the downside beta and the regular beta rather than the upside beta. Furthermore, when stocks are sorted by either the regular beta or the downside beta, it is observed that the standard deviation is higher in the bear market than in the bull market. The observation is consistent with Schwert (1989), who argues that the market is more volatile during recessions.

Overall, table 7.2 suggests that high beta portfolios significantly outperform low beta portfolios in the upside market and underperform them in the downside market. This indicates that the patterns of the return premium are more likely to be captured by a varying risk model that accounts for asymmetric market risk rather than by a constant risk model which assumes symmetric market risk (Bhardwaj and Brooks 1993). Therefore, the next step is to examine whether the varying risk model proposed by Bhardwaj and Brooks (1993) is able to explain the asymmetric risk premium of the asset returns in a mean-variance framework.

Furthermore, table 7.2 confirms the findings of Ang and Chen (2002) that volatility is asymmetric and greater on the downside. As the downside volatility risk appears to be more dominant than the symmetric volatility risk or the upside volatility risk, it is suggested that the asymmetric risk-return relationship is better reflected when portfolios are constructed on the basis of the downside beta compared to those formed on the basis of the regular beta and upside beta. Therefore, the downside beta portfolios are used for regression analyses in subsequent sections and regular/upside beta portfolios are used for robustness tests of the regression results.

7.3.3 Correlation Analysis of the Betas and the Systematic Skewness and the Systematic Kurtosis

Ang and Chen (2002) show that the correlation between asset returns and aggregate market returns is greater in the downside market than in the upside market. To re-examine the findings of Ang and Chen (2002) in the context of the Australian market, this section examines the correlations between five different measures of the return volatility: regular beta, downside beta, upside beta, systematic skewness and systematic kurtosis. The correlation analysis in this section prepares for the subsequent regressions analyses which investigate the risk-return relationship in upside and downside market conditions and examine the roles of systematic skewness and systematic kurtosis in capturing asymmetry in the market risk induced by changes in the market conditions. First, the five measures of return volatility are computed for each stock. Second, correlations of these measures are computed for stocks in each portfolio formed on the basis of different market risk characteristics, such as symmetric, downside and upside risk characteristics. Each of the downside beta portfolios comprises 437 stocks and for each of these stocks, five different measures of the return volatility are computed.

Table 7.3 Correlations between downside beta, regular beta, upside beta, systematic skewness and systematic kurtosis of portfolios formed on the basis of downside beta

The table presents Pearson's correlation coefficients between the betas, the systematic skewness and the systematic kurtosis for stocks of portfolios formed on the basis of the downside beta. Three measures of the betas and two systematic measures of skewness and kurtosis are generated for each stock in the portfolio. Correlations between the downside beta, regular beta, systematic skewness and systematic kurtosis for stocks in each portfolio are computed accordingly. * and ** denote statistical significance at 5 and 1 percent levels for the two-tailed t-test to examine the significance of Pearson's correlation coefficients.

Portfolios sorted by downside beta		β	β^-	β^+	S. Skewness
	β^-	-0.062			
Low β^-	β^+	-0.170*	0.102*		
	S. Skewness	0.121*	0.690**	-0.418**	
	S. Kurtosis	0.153*	0.774**	0.376**	-0.537**
	β^-	-0.038			
2	β^+	-0.631**	0.015		
	S. Skewness	0.4431**	0.222**	-0.830**	
	S. Kurtosis	-0.024	0.227**	0.472**	-0.223**
	β^-	-0.275**			
3	β^+	-0.717**	0.059		
	S. Skewness	0.608**	0.144*	-0.896**	
	S. Kurtosis	-0.099*	0.240**	0.509**	-0.320**
	β^-	-0.173*			
4	β^+	-0.768**	0.145*		
	S. Skewness	0.722**	0.104*	-0.903**	
	S. Kurtosis	-0.165*	0.316*	0.581**	-0.373**
	β^-	-0.312**			
High β^-	β^+	-0.802**	0.323**		
	S. Skewness	0.682**	0.050	-0.859**	
	S. Kurtosis	-0.126*	0.389**	0.500**	-0.255**

Table 7.3 presents correlations between the regular beta, the downside beta, the upside beta, systematic skewness and systematic kurtosis for stocks of portfolios formed on the basis of the downside beta. There are negative correlations between the regular beta and the downside beta on the one hand and the regular beta and the upside betas on the other hand. The correlation between the regular beta and the upside beta is much higher than that between the regular beta and the downside beta. The former correlation is significant in every quintile and ranges from -0.17 to -0.80, while the latter correlation is only significant in three out of five quintiles and ranges from -0.03 to -0.31. While the regular beta can represent the upside beta as it strongly covaries with the upside beta, much of the downside beta is not explained by the regular beta as its correlation with the downside beta is very low. This implies that the downside volatility risk is not fully incorporated in the regular beta. The table shows evidence that the correlations of the regular betas with both the downside and the upside betas are asymmetric and this correlation asymmetry tends to increase when stocks are more exposed to higher volatility risk. Put in another way, riskier stocks will have higher correlation asymmetry. The result, however, does not support the argument of Ang and Chen (2002) that riskier stocks have lower correlation asymmetry.

It is observed that systematic skewness and systematic kurtosis are negatively correlated. However, the degree of this correlation is not high, implying that these factors represent different characteristics of the returns. Systematic skewness has moderate to strong correlations with the regular beta and the upside beta but its correlation with the downside beta is weaker. Systematic kurtosis, on the other hand, has moderate to strong correlations with the downside and the upside beta while its correlation with the regular beta is fairly weak. If systematic skewness and systematic kurtosis are considered as a single risk factor, then this risk factor has a strong correlation with all three measures of the betas. This result is very encouraging and provides some preliminary support to the regression tests of whether systematic skewness and systematic kurtosis effectively capture the asymmetry in the market betas.

Tables 7.4 and 7.5 confirm most of the findings observed in table 7.3. The results in the three tables suggest a weak positive correlation between the downside beta and the upside beta. The correlation between the regular beta and the upside beta is much higher than that between the regular beta and the downside beta. The results obtained from sorting stocks by the downside beta, the regular beta and the upside beta suggest that increasing beta tends to increase correlation asymmetry. The strong correlations between skewness and kurtosis and the three measures of the beta suggest the possibility that these two higher-moment factors can proxy for the asymmetry in the market risk. However, at this stage it is impossible to project to what degree these correlations will affect the return premium in the upside and downside markets or whether systematic skewness and systematic kurtosis can capture the market risk asymmetry. To verify this, a regression analysis is carried out.

Table 7.4 Correlations between downside beta, regular beta, upside beta, systematic skewness and systematic kurtosis for portfolios formed on the basis of regular beta

The table presents Pearson's correlation coefficients between the betas, the systematic skewness and the systematic kurtosis for stocks of portfolios formed on the basis of the regular beta. Three measures of the betas and two systematic measures of skewness and kurtosis are generated for each stock in the portfolio. Correlations between the downside beta, regular beta, systematic skewness and systematic kurtosis for stocks in each portfolio are computed accordingly. * and ** denote statistical significance at 5 and 1 percent levels for the two-tailed t-tests to examine the significance of Pearson's correlation coefficients.

Portfolios sorted by regular beta		**β**	**β^-**	**β^+**	**S. Skewness**
	β^-	-0.080			
Low β	β^+	-0.730**	0.144*		
	S. Skewness	0.662**	0.343**	-0.905**	
	S. Kurtosis	-0.025	0.178*	0.392**	-0.143*
	β^-	-0.143*			
2	β^+	-0.696**	0.060		
	S. Skewness	0.663**	0.168*	-0.892**	
	S. Kurtosis	-0.117*	0.283**	0.661**	-0.406**
	β^-	-0.180**			
3	β^+	-0.722**	-0.033		
	S. Skewness	0.695**	0.187**	-0.839**	
	S. Kurtosis	-0.051	0.238**	0.499**	-0.297**
	β^-	-0.194*			
4	β^+	-0.798**	0.144*		
	S. Skewness	0.703**	0.164*	-0.886**	
	S. Kurtosis	-0.103*	0.338**	0.529**	-0.309**
	β^-	-0.280**			
High β	β^+	-0.785**	0.181**		
	S. Skewness	0.686**	0.080	-0.896**	
	S. Kurtosis	-0.053	0.289**	0.480**	-0.300**

Table 7.5 Correlations between downside beta, regular beta, upside beta, systematic skewness and systematic kurtosis for portfolios formed on the basis of upside beta

The table presents Pearson's correlation coefficients between the betas, the systematic skewness and the systematic kurtosis for stocks of portfolios formed on the basis of the upside beta. Three measures of the betas and two systematic measures of skewness and kurtosis are generated for each stock in the portfolio. Correlations between the downside beta, regular beta, systematic skewness and systematic kurtosis for stocks in each portfolio are computed accordingly. * and ** denote statistical significance at 5 and 1 percent levels for the two-tailed t-tests to examine the significance of Pearson's correlation coefficients.

Portfolios sorted by upside beta		β	β^-	β^+	S. Skewness
Low β^+	β^-	-0.275**			
	β^+	-0.805**	0.142*		
	S. Skewness	0.695**	0.144*	-0.888**	
	S. Kurtosis	-0.067	0.254**	0.418**	-0.172*
2	β^-	-0.147*			
	β^+	-0.643**	0.127*		
	S. Skewness	0.640**	0.088*	-0.873**	
	S. Kurtosis	-0.099*	0.277**	0.521**	-0.308**
3	β^-	-0.222**			
	β^+	-0.676**	0.025		
	S. Skewness	0.539**	0.173*	-0.892**	
	S. Kurtosis	-0.030	0.081	0.504**	-0.320**
4	β^-	-0.081			
	β^+	-0.744**	-0.024		
	S. Skewness	0.743**	0.184**	-0.922**	
	S. Kurtosis	0.021	0.212**	0.428**	-0.204**
High β^+	β^-	-0.217**			
	β^+	-0.791**	0.186**		
	S. Skewness	0.718**	0.047	-0.900**	
	S. Kurtosis	0.006	0.294**	0.366**	-0.177*

7.3.4 The Risk-Return Relationship in Dual- Beta Asset Pricing Model

The preliminary results in the previous sections motivate the study to investigate whether a varying risk model such as the dual-beta asset pricing model proposed by Bhardwaij and Brooks (1993) is sufficient to explain the beta asymmetry in the two-moment framework. The dual-beta asset pricing model is the modified CAPM which includes a binary variable to allow for variations of the market beta across bull and bear markets. In this regression analysis, average excess returns on portfolios formed on the basis of downside, regular and upside betas are used as dependent variables for the dual-beta asset pricing model and the CAPM. This gives perspectives on the range of average returns that competing sets of risk factors must explain.

Tables 7.6, 7.7 and 7.8 present regressions results when dependent variables of the regressions are sorted based on regular, downside and upside betas. The constant market beta of the CAPM is statistically significant at 1 percent in all three tables. High-beta stocks are more sensitive to market risk, which is evidenced by beta coefficients monotonically increasing from the low beta to the high beta portfolios. The result is consistent with the earliest empirical studies of the CAPM such as Black *et al.* (1972), who find high reward for holding higher-beta stocks. However, this evidence does not imply that the CAPM holds, as the CAPM predicts that no variables other than the market beta should explain a firm's expected return.

Table 7.6 Regressions results for two-moment models for portfolios formed on the basis of downside beta for the period of January 1992 to December 2006

The table presents estimates of regressions with Newey-West standard errors for the followings models: (1) The CAPM: $R_t - R_{ft} = \alpha + \beta(R_{mt} - R_{ft})$ and (2) The dual-beta CAPM: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta(R_{mt} - R_{ft}) + \beta' d_t(R_{mt} - R_{ft}) + e_t$ where R_t, R_{ft} and R_{mt} are the return of the portfolio, the risk free-rate and the market return at time t respectively.; d_t is a dummy variable which is equals to 1 in bull markets and 0 in bear markets. Student's t-statistics are presented in parentheses. * and ** denote statistical significance at 5 and 1 percent levels.

	CAPM			Dual-beta CAPM				
	α	**β**	**Adj. R^2**	**α**	**α′**	**β**	**β′**	**Adj. R^2**
Low β^-	-0.0018 (2.96)**	0.626 (17.31)**	0.502	0.0048 (2.77)**	-0.0205 (-7.64)**	0.681 (11.89)**	-0.252 (-3.04)**	0.556
2	-0.0005 (-1.05)	0.659 (21.65)**	0.642	0.0096 (8.14)**	-0.0222 (-12.14)**	0.819 (21.43)**	-0.033 (-0.58)	0.694
3	-0.0006 (-1.15)	0.773 (26.40)**	0.640	0.0084 (6.09)**	-0.0194 (-9.11)**	0.928 (21.09)**	-0.240 (-1.95)*	0.730
4	-0.0017 (-2.05)*	1.052 (19.56)**	0.492	0.0075 (3.32)**	-0.0177 (-5.04)**	1.131 (15.49)**	-0.311 (-1.99)*	0.596
High β^-	-0.0030 (-3.18)**	1.235 (14.57)**	0.393	0.0092 (2.79)**	-0.0189 (-3.69)**	1.431 (13.51)**	-0.522 (-3.31)**	0.489

The hypothesis of whether the beta premium is asymmetric is tested by adding a binary variable to the CAPM to account for the difference in return premium between bull and bear markets. Overall, table 7.6 shows that the premium difference is significant at 5 percent level in four of five quintiles examined. Importantly, the return premium is significantly higher in the bull market than in the bear market as evidenced by the negative sign of the premium difference in all 5 portfolios. This signifies that the market premium is asymmetric and covaries more strongly with market returns during bear markets than during bull markets. The result is consistent with a long literature documenting the increasing of volatility premium when stocks are in the downside market (Schwert (1989) and Campbell and Hentschel (1992)). The results from table 7.7 confirm conclusions drawn from table 7.6 while the results from table 7.8 indicate that beta asymmetry is significant for only high upside beta portfolios.

Table 7.7 Regressions results for two-moment models for portfolios formed on the basis of regular beta for the period of January 1992 to December 2006

The table presents estimates of regressions with Newey-West standard errors for the followings models: (1) The CAPM: $R_t - R_{ft} = \alpha + \beta(R_{mt} - R_{ft})$ and (2) The dual-beta CAPM: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta(R_{mt} - R_{ft}) + \beta' d_t(R_{mt} - R_{ft}) + e_t$ where R_t, R_{ft} and R_{mt} are the return of the portfolio, the risk free-rate and the market return at time t respectively.; d_t is a dummy variable which is equals to 1 in bull markets and 0 bear markets. Student's t-statistics are presented in parentheses. * and ** denote statistical significance at 5 and 1 percent levels.

	CAPM			Dual-beta CAPM				
	α	**β**	**Adj. R^2**	**α**	**α′**	**β**	**β′**	**Adj. R^2**
Low β	0.0010 (-1.65)	0.674 (20.88)**	0.399	0.0134 (9.84)**	-0.0325 (-14.93)**	0.744 (17.31)**	-0.391 (-3.48)**	0.611
2	-0.0008 (-1.56)	0.731 (31.38)**	0.600	0.0116 (9.16)**	-0.0259 (-12.81)**	0.866 (21.73)**	-0.193 (-2.40)**	0.731
3	-0.0012 (-2.24)**	0.881 (34.55)**	0.645	0.0090 (6.25)**	-0.0213 (-9.22)**	0.970 (21.31)**	-0.091 (-1.70)	0.744
4	-0.0030 (-3.77)**	0.964 (28.01)**	0.544	0.0027 (1.18)	-0.0118 (-3.29)**	1.127 (14.43)**	-0.052 (-0.68)	0.617
High β	-0.0047 (-3.81)**	1.181 (22.55)**	0.436	-0.0017 (-0.49)	-0.0055 (-0.97)	1.347 (11.04)**	-0.189 (1.98)*	0.516

Tables 7.6, 7.7 and 7.8 show that the adjusted R-squared increases significantly in every beta quintile when the CAPM allows risk to vary between bull and bear periods. The results are economically significant. In particular, the highest increase in table 7.6 is from 0.393 to 0.489 or a 25% increase of the former adjusted R-squared. In table 7.7, the increase is from 0.399 to 0.611 or more than a 50% increase from the former adjusted R-squared. The significant increase in the adjusted R-squared when the dual-beta CAPM allows the market beta to vary between bull and bear periods and the significance of the return premium difference in most of the portfolio indicate advantages of the dual-beta asset pricing model proposed by Bhardwaij and Brooks (1993) over the CAPM in explaining the variation in expected returns. These findings, however, do not support Fabozzi and Francis (1979), who document changes in systematic risk (beta) and

abnormal return (alpha) over bull and bear markets but fail to prove that these changes are significant.

Table 7.8 Regressions results for two-moment models for portfolios formed on the basis of upside beta for the period of January 1992 to December 2006

The table presents estimates of regressions with Newey-West standard errors for the followings models: (1) The CAPM: $R_t - R_{ft} = \alpha + \beta(R_{mt} - R_{ft})$ and (2) The dual-beta CAPM: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta(R_{mt} - R_{ft}) + \beta' d_t(R_{mt} - R_{ft}) + e_t$ where R_t, R_{ft} and R_{mt} are the return of the portfolio, the risk free-rate and the market return at time t respectively; d_t is a dummy variable which is equals to 1 in bull markets and 0 bear markets. Student's t-statistics are presented in parentheses. * and ** denote statistical significance at 5 and 1 percent levels.

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	CAPM			Dual-beta CAPM				
	α	**β**	**Adj. R^2**	**α**	**α**	**β**	**Adj. R^2**	
Low β^+	-0.0023 (-2.87)**	0.747 (20.92)**	0.400	0.0065 (2.88)**	-0.0125 (-3.44)**	0.914 (12.76)**	0.013 (0.19)	0.455
2	-0.0013 (-2.22)**	0.796 (30.21)**	0.581	0.0090 (5.52)**	-0.0188 (-7.25)**	0.933 (18.20)**	-0.034 (-0.54)	0.654
3	-0.0007 (-1.44)	0.815 (35.38)**	0.656	0.0100 (7.65)**	-0.0216 (-10.33)**	0.928 (22.42)**	-0.072 (-1.01)	0.753
4	-0.0017 (-2.87)**	0.873 (33.09)**	0.625	0.0076 (4.82)**	-0.0202 (-8.01)**	1.085 (19.82)**	-0.117 (-2.05)*	0.726
High β^+	-0.0044 (-4.68)**	0.916 (24.96)**	0.487	0.0025 0.96	-0.0220 (-5.19)**	1.179 (12.89)**	-0.195 (-2.20)*	0.592

To sum up, the regression results on the CAPM versus the dual-beta asset pricing model imply that the market beta is unstable and therefore question the appropriateness of the CAPM to explain variation of asset returns. The results support the dual-beta asset-pricing model proposed by Bhardwaj and Brooks (1993) which suggests the need to include varying risk measures to incorporate changes in market conditions. The regression results of this model suggest that the downside risk and the upside risk are priced asymmetrically and, most importantly, the required risk premium for the downside risk is significantly higher than the premium for the upside risk. This explains the behavioural framework of Kahneman and Tversky's (1979) loss aversion preferences which suggests that investors place greater weight on losses relative to gains in their utility functions. Hence, they demand a greater compensation, in the form of higher expected returns, for holding stocks with high downside risk. The evidence that the return premium is much higher in the downside risk is consistent with either a transaction cost explanation or a recession-related premium. Karpoff (1997) argues that higher transaction costs in economic recessions contribute to higher return premium in the downturn. Schwert (1989) finds greater market volatility during recessions, which could contribute to greater bid-ask spreads leading to higher trader's required premium to compensate for greater uncertainty. The results also reveal that stocks exposed to higher volatility risk tend to have higher premium asymmetry. Finally, the results are consistent with Kothari, Shanken and Sloan (1995) and Howton and Peterson (1998), who show that the importance of beta is sensitive to the way it is estimated. Having examined the similarities and differences of the findings when the market beta is constructed in various ways to reflect the market conditions, the study concludes that the downside risk appears to be more important to investors than the upside risk, and therefore it makes more sense to use downside beta portfolios to examine the risk-return relationship. The results suggest that the effectiveness of the dual-beta asset pricing model is improved when dependent portfolios are formed on the basis of downside beta.

As the preliminary results have documented strong correlations between systematic skewness and systematic kurtosis and upside and downside betas, the next step in this regression analysis is to examine whether this market beta asymmetry still persists when systematic skewness and systematic kurtosis are added to the dual-beta asset pricing model.

7.3.5 Can Systematic Skewness and Systematic Kurtosis Proxy for Market Risk Asymmetry?

It is documented in the previous section that market risk asymmetry is significant under the two-moment framework; however, is it still significant under the four-moment framework? Because systematic skewness and systematic kurtosis are associated with non-normalities of asset returns, there is a possibility that they have a close relationship with beta asymmetry. This study tests whether the asymmetry in the risk premium documented in the previous still persists when the asset-pricing model incorporates systematic skewness and systematic kurtosis. That is, the study investigates whether systematic skewness and systematic kurtosis can effectively capture the asymmetric risk caused by changes in the market conditions. The hypothesis is tested using the four-moment model versus dual-beta four-moment models. Similarly to the analysis in the previous section, binary variables, which take a unitary value in bull periods and zero otherwise, are used to measure the premium differences of the market and the systematic skewness and the systematic kurtosis risk factors in both bull and bear markets. The results of this test are presented in tables 7.9, 7.10 and 7.11.

Tables 7.9, 7.10 and 7.11 present the effects of systematic skewness and systematic kurtosis on the beta asymmetry for portfolios sorted by downside, regular and upside betas. Time-series regressions with Newey-West standard errors are first run with the four-moment model (Model 1). When the market beta is no longer the sole independent variable, the tables show that the market coefficient still retains its significance. Importantly, regression results generated from model 1 show that the systematic skewness and the systematic kurtosis factors are significant in explaining the variation in expected returns. However, skewness effects tend to be more significant than kurtosis effects for the Australian stocks. This is consistent with the study of Doan *et al.* (2010) which documents that skewness plays a more important role in Australian returns while kurtosis has more dominant influence on U.S. stock returns.

In model 2, the influence of bull and bear markets on the market premium is examined in a four-moment framework. In this model, binary variables are added to the model to account for differences in the intercepts and in the market betas between bull and bear markets. The regression results from this model show that the significance of the market risk, the systematic skewness and the systematic kurtosis risk factor loadings do not change much from the results generated from model 1.

To investigate whether the market beta asymmetry documented in the two-moment framework still persists in the four-moment framework, the study compares the regression results achieved from this model to those from tables 7.6, 7.7 and 7.8. The following important points stand out. When systematic skewness and systematic kurtosis are added to the dual-beta asset pricing model, the premium difference of the market risk between bull and bear markets becomes insignificant in every portfolio examined while systematic skewness and systematic kurtosis are statistically important in most portfolios. This implies that systematic skewness and systematic kurtosis are able to capture the asymmetry in the risk factor loading caused by changes in the market conditions. The regression results in tables 7.6, 7.7 and 7.8 suggest that in the two-moment framework the dual-beta asset pricing model proposed by Bhardwaj and Brooks (1993) can account for market risk asymmetry between bull and bear markets. However, the results in tables 7.9, 7.10 and 7.11 suggest that in the four-moment framework, systematic skewness and systematic kurtosis can also account for this asymmetry between bull and bear markets. Since the usefulness of the dual-beta asset pricing model is sensitive to the way bull and bear periods are constructed, the four-moment model has advantages over the dual-beta model because it provides an effective method to capture the beta asymmetry without having to assume a specific market condition construction.

Model 3 tests whether changes in market conditions have any effects on the skewness and kurtosis factor loadings. The regression results generated from the model show that the effects of systematic skewness and systematic kurtosis on expected returns are not subject to market conditions, except for the highest beta portfolio. This is evidenced by that the significance of these factors do not change substantially from model 1 to model 3 and the coefficients of the dummy variables of these factors, which represent premium differences between bull and bear markets, are not statistically significant, except for the highest beta portfolio. Overall, the regression results from model 3 suggest systematic skewness and systematic kurtosis can be used as a proxy for the beta asymmetry caused by changes in the market conditions.

Table 7.9 Impacts of systematic skewness and systematic kurtosis on beta asymmetry: A time-series analysis for portfolios sorted by downside beta for the period of January 1992 to December 2006

The table presents estimates of regressions with Newey-West standard errors for the followings models:

Model 1: $R_t - R_{ft} = \alpha + \beta_1(R_{mt} - R_{ft}) + \beta_2 S_t + \beta_3 K_t + e_t$

Model 2: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta_1(R_{mt} - R_{ft}) + \beta'_1 d_t(R_{mt} - R_{ft}) + \beta_2 S_t + \beta_3 K_t + e_t$

Model 3: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta_1(R_{mt} - R_{ft}) + \beta'_1 d_t(R_{mt} - R_{ft}) + \beta_2 S_t + \beta'_2 d_t S_t + \beta_3 K_t + \beta'_3 d_t K_t + e_t$

where R_t , R_{ft} and R_{mt} are the return of the portfolio, the risk free-rate and the market return respectively, S_t and K_t are the systematic skewness and kurtosis factors respectively and d_t is a dummy variable which is equals to 1 in bull markets and 0 in bear markets. Student's t-statistics are presented in parentheses. * and ** denote statistical significance at 5 and 1 percent levels respectively.

Model	Portfolio	α	α'	β_1	β'_1	β_2	β'_2	β_3	β'_3	Adj-R2
Model 1	Low β^-	-0.0185		0.7665		-0.1780		-0.1988		0.555
		(-3.22)**		(20.79)**		(-3.06)**		(-3.04)**		
	2	-0.0004		0.7731		-0.1724		-0.1547		0.694
		(-1.01)		23.93)**		(-3.82)**		(-3.29)**		
	3	-0.0032		0.7938		-0.3001		0.0735		0.712
		(-0.67)		(28.03)**		(-6.41)**		(1.52)		
	4	-0.0008		0.8714		-0.5733		0.0930		0.745
		(-1.45)		(20.67)**		(-10.44)**		(1.41)		
	High β^-	-0.0022		0.8906		-0.8979		0.1900		0.817
		(-3.21)**		(21.02)**		(-14.04)**		(2.60)**		
Model 2	Low β^-	0.0045	-0.0194	0.7035	0.1740	-0.1921		-0.2380		0.574
		(2.09)*	(-6.98)**	(8.54)**	(1.74)	(-2.73)**		(2.78)*		
	2	0.0095	-0.0217	0.8234	0.0685	-0.1667		-0.16638		0.755
		(7.12)**	(-11.30)**	(14.05)**	(0.95)	(-2.89)**		(2.75)**		
	3	0.0087	-0.0205	0.84400	0.1079	-0.3245		0.1625		0.782
		(5.86)*	(-9.92)**	(15.64)**	(1.60)	(-5.70)**		(2.21)*		
	4	0.0092	-0.0228	0.9025	0.1765	-0.5990		0.1338		0.794
		(4.23)**	(-7.98)**	(10.16)**	(1.83)	(-8.67)**		(1.31)		
	High β^-	0.0122	-0.0284	0.9849	0.1458	-0.9648		0.2136		0.861
		(6.50)**	(-10.38)**	(12.04)**	(1.59)	(-11.56)**		(2.01)**		
Model 3	Low β^-	0.0002	-0.0205	0.7441	0.0232	-0.2698	-0.0052	-0.3746	0.2571	0.587
		(0.97)	(-8.03)**	(9.09)*	(0.42)	(-2.72)**	(-0.04)	(2.17)*	(1.41)	
	2	0.0092	-0.0227	0.8546	0.0460	-0.1321	-0.1056	-0.1909	0.0770	0.767
		(7.00)**	(13.17)**	(14.12)**	(0.63)	(-1.98)*	(-1.33)	(1.67)	(0.63)	
	3	0.0085	-0.0215	0.8765	-0.0735	-0.3513	-0.0688	0.1814	0.1600	0.790
		(5.67)**	(-11.57)**	(16.35)**	(-1.12)	(-4.76)**	(-0.69)	(1.97)*	(1.29)	
	4	0.0088	-0.0241	0.9543	-0.1074	-0.7220	-0.0370	0.1608	0.2585	0.806
		(4.14)**	(-9.64)**	(11.43)**	(1.20)	(-7.79)**	(0.29)	(1.83)	(1.45)	
	High β^-	0.0118	-0.0293	1.0534	-0.0451	-1.2737	0.3823	0.2880	0.5921	0.870
		(6.51)**	(-11.97)**	(14.08)**	(-0.54)	(-10.53)**	(2.85)**	(2.04)*	(3.42)**	

Table 7.10 Impacts of systematic skewness and systematic kurtosis on beta asymmetry: A time-series analysis for portfolios sorted by regular beta for the period of January 1992 to December 2006

The table presents estimates of regressions with Newey-West standard errors for the followings models:

Model 1: $R_t - R_{ft} = \alpha + \beta_1(R_{mt} - R_{ft}) + \beta_2 S_t + \beta_3 K_t + e_t$

Model 2: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta_1(R_{mt} - R_{ft}) + \beta'_1 d_t(R_{mt} - R_{ft}) + \beta_2 S_t + \beta_3 K_t + e_t$

Model 3: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta_1(R_{mt} - R_{ft}) + \beta'_1 d_t(R_{mt} - R_{ft}) + \beta_2 S_t + \beta'_2 d_t S_t + \beta_3 K_t + \beta'_3 d_t K_t + e_t$

where R_t , R_{ft} and R_{mt} are the return of the portfolio, the risk free-rate and the market return respectively, S_t and K_t are the systematic skewness and kurtosis factors respectively and d_t is a dummy variable which is equals to 1 in bull markets and 0 in bear markets. Student's t-statistics are presented in parentheses. * and ** denote statistical significance at 5 and 1 percent levels respectively.

Model		α		β_1		β_2		β_3		Adj-R2
1	Low β	-0.0010 (-1.86)		0.758 (18.14)**		-0.402 (-7.61)**		-0.430 (-8.68)**		0.585
	2	-0.0007 (-1.44)		0.854 (22.97)**		-0.202 (-4.10)**		-0.171 (-3.21)**		0.693
	3	-0.0009 (-1.66)		0.815 (24.24)**		-0.282 (-4.99)**		0.064 (1.26)		0.710
	4	-0.0018 (-3.04)**		0.907 (21.75)**		-0.512 (-8.22)**		0.103 (1.59)		0.739
	High β	-0.0024 (-3.44)**		1.0740 (22.75)**		-0.766 (-9.55)**		0.345 (3.62)**		0.809
Model		α	α'	β_1	β'_1	β_2		β_3		Adj-R2
2	Low β	0.0129 (10.24)**	-0.0312 (-15.35)**	0.802 (14.25)**	0.080 (1.09)	-0.352 (-5.70)**		-0.396 (-6.88)**		0.687
	2	0.0114 (8.36)**	-0.0255 (-13.44)**	0.881 (16.28)**	0.004 (0.06)	-0.162 (-2.85)**		-0.162 (-2.40)**		0.740
	3	0.0094 (5.76)**	-0.0222 (-9.58)**	0.910 (14.46)**	0.037 (0.45)	-0.260 (-3.80)**		0.014 (1.15)		0.766
	4	0.0046 (2.07)*	-0.0162 (-5.58)**	0.860 (8.82)**	0.185 (1.77)	-0.515 (-6.23)**		0.124 (1.52)		0.772
	High β	0.0025 (0.97)	-0.0148 (-4.41)**	0.985 (7.79)**	0.354 (3.02)**	-0.786 (-7.09)**		0.303 (2.27)*		0.838
Model		α	α'	β_1	β'_1	β_2	β'_2	β_3	β'_3	Adj-R2
3	Low β	0.0127 (10.28)**	-0.0318 (-16.50)**	0.831 (14.12)**	0.043 (0.56)	-0.374 (-3.86)**	-0.084 (-0.71)	-0.465 (-4.66)**	0.135 (1.21)	0.699
	2	0.0112 (8.07)**	-0.0260 (-14.45)**	0.905 (17.12)**	-0.024 (-0.39)	-0.172 (-1.71)	-0.086 (-0.68)	-0.212 (-2.63)**	0.100 (0.72)	0.746
	3	0.0092 (5.54)**	-0.0228 (-10.41)**	0.942 (15.38)**	-0.007 (-0.09)	-0.303 (-2.72)**	-0.057 (-0.42)	0.021 (1.14)	0.175 (1.18)	0.774
	4	0.0042 (1.91)	-0.0169 (-6.52)**	0.912 (9.92)**	0.084 (0.89)	-0.742 (-4.75)**	0.219 (1.29)	0.055 (1.31)	0.181 (1.37)	0.785
	High β	0.0019 (0.77)	-0.0157 (-5.29)**	1.075 (9.17)**	0.223 (2.25)*	-1.086 (-6.07)**	0.301 (1.46)	0.262 (2.17)*	0.630 (2.57)**	0.849

Table 7.11 Impacts of systematic skewness and systematic kurtosis on beta asymmetry: A time-series analysis for portfolios sorted by upside beta for the period of January 1992 to December 2006

The table presents estimates of regressions with Newey-West standard errors for the followings models:

Model 1: $R_t - R_{ft} = \alpha + \beta_1(R_{mt} - R_{ft}) + \beta_2 S_t + \beta_3 K_t + e_t$

Model 2: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta_1(R_{mt} - R_{ft}) + \beta'_1 d_t(R_{mt} - R_{ft}) + \beta_2 S_t + \beta_3 K_t + e_t$

Model 3: $R_t - R_{ft} = \alpha + \alpha' d_t + \beta_1(R_{mt} - R_{ft}) + \beta'_1 d_t(R_{mt} - R_{ft}) + \beta_2 S_t + \beta'_2 d_t S_t + \beta_3 K_t + \beta'_3 d_t K_t + e_t$

where R_t, R_{ft} and R_{mt} are the return of the portfolio, the risk free-rate and the market return respectively, S_t and K_t are the systematic skewness and kurtosis factors respectively and d_t is a dummy variable which is equals to 1 in bull markets and 0 in bear markets. Student's t-statistics are presented in parentheses. * and ** denote statistical significance at 5 and 1 percent levels respectively.

Model 1		α	β_1	β_2	β_3	Adj-R2
	Low β^+	-0.0013	0.717	-0.938	-0.477	0.644
		(-2.11)*	(18.69)**	(-14.01)**	(-7.32)**	
	2	-0.0007	0.758	-0.484	-0.195	0.682
		(-1.37)	(21.99)**	(-9.04)**	(-3.42)**	
	3	-0.0004	0.797	-0.318	-0.166	0.693
		(-0.85)	(23.35)**	(-6.09)**	(-3.16)**	
	4	-0.0011	0.808	-0.288	-0.014	0.688
		(-2.11)*	(23.76)**	(-5.16)**	(-0.25)	
	High β^+	-0.0030	0.868	-0.268	0.418	0.719
		(-4.45)*	(21.34)**	(-3.49)**	(4.36)**	

Model 2		α	α'	β_1	β'_1	β_2	β_3	Adj-R2
	Low β^+	0.0076	-0.0146	0.755	-0.016	-0.986	-0.557	0.679
		(4.12)**	(-5.22)**	(9.35)**	(-0.16)	(-12.17)**	(-6.83)**	
	2	0.0098	-0.0207	0.810	0.013	-0.472	-0.186	0.742
		(6.53)**	(-9.33)**	(12.81)**	(0.15)	(-6.80)**	(-2.45)**	
	3	0.0104	-0.0223	0.880	0.019	-0.287	0.158	0.778
		(6.83)**	(-11.09)**	(13.36)**	(0.26)	(-4.50)**	(2.36)*	
	4	0.0084	-0.0219	0.874	0.085	-0.258	0.026	0767
		(4.64)**	(-9.16)**	(12.62)**	(1.01)	(-3.66)**	(0.34)	
	High β^+	0.0053	-0.0281	0.917	0.467	-0.240	0.419	0.788
		(2.21)**	(-8.93)**	(8.43)**	(4.18)**	(-2.35)*	(3.09)**	

Model 3		α	α'	β_1	β'_1	β_2	β'_2	β_3	β'_3	Adj-R2
	Low β^+	0.0074	-0.0151	0.777	-0.037	-0.966	-0.134	-0.579	0.054	0.681
		(3.97)**	(-5.48)*	(9.23)**	(-0.34)	(-7.08)**	(-0.77)	(-3.76)**	(0.32)	
	2	0.0096	-0.0213	0.842	-0.025	-0.489	-0.103	-0.256	0.138	0.751
		(6.36)**	(-10.21)**	(13.08)**	(-0.31)	(-4.23)**	(-0.73)	(-1.96)**	(0.92)	
	3	0.0101	-0.0228	0.906	-0.021	-0.343	-0.005	0.152	0.170	0.784
		(6.72)**	(-12.09)**	(13.68)**	(-0.28)	(-2.82)**	(-0.04)	(1.65)	(1.19)	
	4	0.0081	-0.0226	0.913	0.026	-0.370	0.052	0.187	0.286	0.774
		(4.50)**	(-10.44)**	(13.37)**	(0.26)	(-2.92)**	(0.37)	(1.22)	(1.70)	
	High β^+	0.0047	-0.0289	0.985	0.315	-0.638	0.482	0.333	0.766	0.810
		(2.04)*	(-10.62)**	(10.62)**	(3.67)**	(-4.03)**	(2.63)**	(2.15)*	(3.27)**	

Table 7.12 Impacts of systematic skewness and systematic kurtosis on asset returns under bull and bear market conditions: An analysis using F-tests

The table presents F-tests for the following hypotheses:

H1: $\beta'_2 = \beta'_3 = 0$, against the alternative hypothesis H1a: $\beta'_2 \neq 0$ or $\beta'_3 \neq 0$,

H2: $\beta'_1 = \beta'_2 = \beta'_3 = 0$,against the alternative hypothesis H2a: $\beta'_1 \neq 0$ or $\beta'_2 \neq 0$ or $\beta'_3 \neq 0$,

where estimates of the intercept, the market, the systematic skewness and systematic kurtosis factors are generated by regressions as illustrated in tables 7.9,7.10 and 7.11.

P-values are presented in parentheses. Student's t-statistics are presented in parentheses. * and ** denote statistical significance at 5 and 1 percent levels respectively.

Portfolios	**H1**	**H2**
	F-statistics (P-value)	**F-statistics (P-value)**
Low β^-	2.07 (0.130)	2.01 (0.115)
2	2.05 (0.132)	1.95 (0.124)
3	2.12 (0.124)	2.08 (0.105)
4	2.51 (0.085)	2.46 (0.065)
High β^-	5.25 (0.006)*	4.02 (0.010)**
Low β	2.20 (0.114)	1.98 (0.119)
2	2.12 (0.122)	1.42 (0.239)
3	2.25 (0.108)	2.01 (0.114)
4	2.43 (0.091)	2.30 (0.079)
High β	5.72 (0.004)**	5.50 (0.001)**
Low β^+	1.03 (0.360)	0.690 (0.503)
2	1.85 (0.159)	1.71 (0.184)
3	2.13 (0.122)	1.570 (0.211)
4	2.32 (0.101)	2.21 (0.113)
High β^+	8.44 (0.000)**	9.23 (0.000)**

The appropriateness of the four-moment model over the dual-beta four-moment models can be tested using F-tests. The following hypotheses are tested:

(1) the effects of systematic skewness and systematic kurtosis do not change in response to changes in the market conditions; and

(2) once these factors are added into the CAPM, the changes in the market conditions do not have any significant effects on the explanatory variables of the four-moment model, i.e. the market premium, the skewness premium and the kurtosis premium.

Table 7.12 presents F-test results when dependent variables are portfolios formed on the basis of downside, regular and upside betas. The results in table 7.12 indicate that these hypotheses are not rejected in four of five quintile portfolios and they are only rejected for the highest beta quintile. This shows that the four-moment model has advantages over the dual-beta asset four-moment pricing models.

7.4 Conclusions

This chapter constructs beta measures in bull and bear markets to examine the beta asymmetry. The results reveal that in the two-moment framework, the downside risk and the upside risk are priced asymmetrically and the return premium for the downside risk is significantly higher than the premium for the upside risk. The study uses systematic measures of skewness and kurtosis which are formulated as analogs of the market beta in to establish a link between these factors and market risk asymmetry. The main finding of this study is that in the four-moment framework, systematic skewness and systematic kurtosis can capture the beta asymmetry effectively. As the evidence of risk asymmetry generated from the dual-beta asset pricing model proposed by Bhardwaj and Brooks (1993) is sensitive to the way bull and bear periods are constructed, the main finding also implies that the four-moment model provides a convenient method to capture the asymmetry in risk factor loadings and therefore has advantages over the dual-beta asset-pricing model proposed by Bhardwaj and Brooks (1993).

Finally, the overall regression results presented in this chapter suggest that the asymmetric risk-return relationship and the link between this asymmetry and systematic skewness and systematic kurtosis are better revealed when dependent portfolios are formed on the basis of the downside beta. Due to the lack of Australian studies in this area, the findings in this study advocate further research on the implications of skewness and kurtosis in portfolio allocation and risk management.

CHAPTER 8. CONCLUSIONS

8.1 Introduction

The foundation of portfolio theory and capital asset pricing models (CAPM) by Markowitz (1952), Sharpe (1964) and Lintner (1965) have led to numerous studies in asset allocation based upon the first two moments of the return distribution. In the mean-variance framework, asset returns are assumed to be normally distributed. However, it has been well documented that asset returns are driven by asymmetric fat-tailed distributions and extreme returns occur too often to be consistent with the normality assumption. For example, Samuelson (1970) and Rubinstein (1973) argue that higher moments are relevant to the investor's decision unless the asset returns are normally distributed or the investor's utility functions are quadratic. Several empirical tests on Sharpe's CAPM (1964) have largely rejected the validity of the model which assumes that an investor's utility function is quadratic and that the co-movement with the market return is the only important factor in pricing assets.

While the non-normalities of asset returns are documented, several empirical studies of CAPM have also pointed out that investors care about downside losses and upside gains asymmetrically. For example, the studies of Levy (1971), Klemkosky and Martin (1975), Levy (1977), Fabozzi and Francis (1977, 1979) and Bhardwaj and Brooks (1993) pose questions on the validity of the Sharpe-Litner CAPM. They argue that the variation in the market risk factor due to premium changes between bull and bear markets is not explained by the Sharpe-Litner CAPM. More recently, the studies of Campbell *et al.* (2001), Bekaert and Harvey (2000) and

Ang and Chen (2002, 2007) document asymmetry between upside and downside market betas. Pettengill *et al.* (1995) propose that if there is no systematic relationship between asset returns and the market beta, continued reliance on the beta as a measure of risk is inappropriate.

Given that the empirical stock return distribution is observed to be asymmetric and leptokurtic and investors' expected returns are asymmetric across bull and bear markets, this study investigated the following research questions:

1. Are asset returns mean-variance efficient?

2. If they are not mean-variance efficient, are they mean-variance-skewness-kurtosis efficient?

3. Are systematic skewness and systematic kurtosis important priced factors for asset returns?

4. Is market risk asymmetric between bull and bear markets? If it is, can systematic skewness and systematic kurtosis proxy for risk asymmetry caused by changes in market conditions?

8.2 Findings

The focus of this study is the relevance of systematic skewness and systematic kurtosis in asset pricing. Four different aspects of this central issue were addressed in the four research questions of this study. These questions were examined in four chapters of empirical analysis.

Chapter 3 provided details of summary statistics of Australian stocks from 1992 to 2009. Overall, the summary statistics showed that returns of Australian stocks do not conform to a normal distribution and that the impact of systematic skewness and systematic kurtosis is present in the data. The descriptive evidence that Australian returns are generally negatively skewed and leptokurtic motivated the study in the subsequent chapters, to find solid statistical evidence that mean and variance are not sufficient to describe asset returns and that skewness and kurtosis add significant explanatory power to the CAPM.

To gain more insight into the empirical distribution of asset returns, mean-variance efficiency tests of asset returns were conducted in Chapter 4. The analysis reported in the chapter was carried out using standard parametric and using nonparametric approaches. With the standard parametric approach, the Wald test and the GRS test were used to test if multivariate intercepts generated from the CAPM were jointly equal to zero. The results generated from both tests firmly rejected the hypothesis of jointly zero intercepts. This implies that asset returns are not mean-variance efficient. As the Wald and GRS tests are based on the assumption of multivariate normal errors, a bootstrap approach was carried out as a robustness test. The purpose of the robustness test was to confirm whether conclusions drawn from the parametric tests are still persistent when the normality assumption is violated. Using the bootstrap method, it was found that the Wald test is largely influenced by the non-normality properties of the errors and therefore its bootstrapped results were not consistent with the results generated using the normality assumption. This is because the Wald-statistic is very sensitive to the distribution of the error term and therefore inferences drawn from it can be seriously misleading if the error term departs from normal distribution (Gibbons, Ross and Shanken 1989). On the other hand, results for the GRS test were consistent even when the assumption of normal errors is violated. This is because the GRS-statistic follows a non-central F-distribution and F-tests are fairly robust when the sample size is large and the error distribution is not normal (MacKinlay 1985).

As skewness and kurtosis are associated with non-normality in asset returns and given that asset returns are not found to be mean-variance efficient, an investigation of whether asset returns are mean-variance-skewness-kurtosis efficient was conducted in Chapter 5. A four-moment model was constructed to test the explanatory power of systematic skewness and systematic kurtosis in explaining patterns of asset returns. An examination of the pricing error from the model using the generalised multivariate approach proposed by Gibbons, Ross and Shanken (1992) showed that systematic skewness and systematic kurtosis can explain the pricing error of the CAPM. Furthermore, the test indicates that the four-moment model was sufficient to explain patterns of asset returns. Most importantly, it was found that asset returns are generally mean-variance-skewness-kurtosis efficient.

The time-series regression approach of Fama and French (1992) was used to examine the sensitivity of expected returns to the existence of systematic skewness and systematic kurtosis at both aggregate and industry level in Chapter 5. The overall results showed that systematic skewness and systematic kurtosis contribute significantly to modelling the volatility of stock returns at both aggregate and industry levels. However, the degree of statistical significance depends on stock characteristics and market conditions. In particular, the roles of systematic skewness and systematic kurtosis in explaining variation in expected returns are more significant in the downside market than in the upside market. At the industry level, it was found that cyclical sectors with volatile cash flows and high leverage, such as materials, industrials and information technology sectors, are more susceptible to market conditions and therefore to the skewness risk. On the other hand, growth sectors which rely heavily on the present value of future growth opportunities, such as industrials, telecommunication, property, consumer discretionary and health care sectors, are more vulnerable to external shocks and therefore to kurtosis risk.

A further investigation to explore the importance of systematic skewness and systematic kurtosis to asset pricing was undertaken in Chapter 6. The question of whether systematic skewness and systematic kurtosis are important pricing factors for asset returns in cross-section was addressed in this chapter. The Fama and MacBeth (1973) methodology was applied to the four-moment model with the finding that systematic skewness and systematic kurtosis command significant risk premiums and therefore they are pricing factors for asset returns. Importantly, when the CAPM is extended to include systematic skewness and systematic kurtosis, these factors appear to be the dominant explanatory variables and the market factor is generally insignificant.

Despite the fundamental role played by the Fama and MacBeth (1973) methodology in modern asset pricing, the method has been criticised for the use of beta estimates in the second pass of the method. Beta estimation is subject to the errors-in-variables (EIV) problem which may lead to the inconsistency of OLS estimators and so the significance of the explanatory power of the four-moment model is subject to question. To assess the impact of the EIV problem, Chapter 6 included analysis based on two alternative approaches. The Shanken (1992) approach derives an adjusted covariance matrix of regressors when the sample size approaches infinity while the Dagenais and Dagenais (1997) approach generates parameters proxying for the difference between the true value of betas and their estimates. The results from these two approaches showed that the significance of the market and the systematic skewness and systematic kurtosis premiums measured by the traditional Fama and MacBeth cross-sectional regressions are overstated. Nevertheless systematic skewness and systematic kurtosis still retain their significance as pricing factors for cross-sectional asset returns for the 1992–2009 period.

It has been argued that investors care differently about downside losses versus upside gains in an absolute sense. The analysis reported in Chapter 7, examined how an investor's expected return varies under different market conditions. Together with systematic measures of skewness and kurtosis, risk-varying betas were developed for measuring, comparing and testing market risk asymmetry. Using the dual-beta asset-pricing model proposed by Bhardwaj and Brooks (1993) for the time-series analysis, it was found that in the two-moment framework, the downside risk and the upside risk are priced asymmetrically and the market risk premium for the downside risk is significantly larger than the premium for the upside risk.

An investigation to establish a link between market risk asymmetry and systematic skewness and systematic kurtosis was reported in Chapter 7 which investigated whether the market beta asymmetry documented in the two-moment framework still persists if the asset pricing model incorporates systematic skewness and systematic kurtosis. It was found that in the four-moment framework, systematic skewness and systematic kurtosis capture the asymmetry in risk factor loadings. As the evidence of risk asymmetry generated from the dual-beta model is sensitive to the way bull and bear periods are constructed, the finding implies that the four-moment model has advantages over the dual-beta asset-pricing model proposed by Bhardwaj and Brooks (1993). Finally, it was found that the asymmetric risk-return relationship and the link between this asymmetry and systematic skewness and systematic kurtosis are better revealed when examined portfolios are formed on the basis of the downside beta.

8.3 Key Contributions

Although the topic of skewness and kurtosis has been mentioned widely in the literature, to the author's knowledge, this study is the first one to directly examine the roles of both skewness and kurtosis in asset pricing using multivariate regression and bootstrapping approaches. The main contributions of this study can be summarised in three key findings. First, the study is the first one to conclude that Australian stocks are generally mean-variance-skewness-kurtosis efficient rather than mean-variance efficient. Second, systematic skewness and systematic kurtosis are important pricing factors for asset returns. Third, this is the first study in the literature to successfully show that systematic skewness and systematic kurtosis capture market risk asymmetry and thus provide proxies for the beta asymmetry.

Other important findings in this study are:

- Australian stocks are generally more sensitive to skewness risk than kurtosis risk.
- Cyclical stocks, which are more susceptible to the market conditions, are more likely exposed to skewness risk while growth stocks, which react vigorously to external shocks, are more vulnerable to kurtosis risk.
- When the CAPM is extended to include systematic skewness and systematic kurtosis, these factors appear to be the dominant explanatory variables and make market factor insignificant.
- Downside risk and upside risk are priced asymmetrically and the market return premium for downside risk is significantly higher than the premium for upside risk.

- The four-moment model developed in this study has advantages over the dual-beta asset pricing model proposed by Bhardwaj and Brooks (1993) because the evidence of risk asymmetry documented by using the dual-beta model is sensitive to the way bull and bear periods are specified.
- The asymmetric risk-return relationship and the link between this asymmetry and systematic skewness and systematic kurtosis are better revealed when examined portfolios are formed on the basis of the downside beta.

An incidental finding was that the GRS test has superior performance to the Wald test with this type of return data.

8.4 Directions for Future Research

Asset pricing in the four-moment framework, which is the focus of this study, is a broad area with numerous implications in finance. The main finding of the study, that asset returns are generally mean-variance-skewness-kurtosis efficient rather than conventionally mean-variance efficient, suggests that portfolio optimisation and performance measurement should be based on the four-moment framework rather than on the conventional mean-variance framework. However, studies on these topics within the four-moment framework are often limited and approaches towards them are varied. Studies on performance measurement focus on overcoming the shortcoming of mean-variance measures by using downside measures such as the semi-variance or the second-order lower partial moment rather than by incorporating skewness and kurtosis in the measures. Similarly, studies of portfolio optimisation are mostly constructed on the basis of mean-variance dimensions. However, as this study points out, skewness and kurtosis are important to the investor's decision and so portfolio optimisation should span four dimensions of mean-variance-skewness-kurtosis.

To incorporate skewness and kurtosis into performance measures, it is necessary to develop a general function of risk measures which incorporates the variance, skewness and kurtosis of the asset returns. This function will also need to reflect the characteristics of the investor's utility function. For example, Prakash and Bear (1986) developed the composite Treynor measure incorporating skewness for investors who have a power utility function as follows:

$$\lambda_2 = \frac{E(R_i) - R_f}{\eta \mathrm{Cov}(R_i, R_m) + E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^2)},$$

where $\eta = -\frac{2b}{b+1}[a + E(R_m)]$ where a, b are the parameters of a power utility function as:

$$U(\widetilde{W}) = (b-1)^{-1}(a + \widetilde{W})^{1-1/b},$$

and λ_2 is the modified Treynor ratio for skewedly distributed asset returns,

(see Prakash and Bear (1986) for details).

Based on the approach proposed by Prakash and Bear (1986), the development of a modified Treynor ratio to incorporate the kurtosis effect for leptokurtic asset returns is one avenue for future research arising from this study. The research may provide more accurate measures for asset performance than existing performance measures.

Given the importance of skewness and kurtosis documented in this study, it is desirable to incorporate these factors into portfolio optimisation. Traditionally, portfolio optimisation only spans two dimensions, mean and variance, as follows:

1. Maximise mean function: $E(R) = X^t(\overline{R} - \overline{R}_f)$, and
2. Minimise variance function: $V(X) = E(X^t(R - \overline{R}))^2$,

where: $X^t = [x_1, x_2, .., x_n]$ is the transpose weight vector of assets in the portfolio and $\overline{R}^t = (\overline{R}_1, \overline{R}_2, .., \overline{R}_n)$ where R_i is the return of asset i.

With the presence of skewness and kurtosis, portfolio optimisation involves a trade-off between two set of objectives: (1) maximising expected return and positive skewness; and (2) minimising the variance and kurtosis as follows:

1. Maximise mean function: $E(R) = X^t(\overline{R} - \overline{R}_f)$,
2. Minimise variance function : $V(X) = E(X^t(R - \overline{R}))^2$,
3. Maximise skewness function: $S(X) = E(X^t(R - \overline{R}))^3$, and
4. Minimise kurtosis function: $K(X) = E(X^t(R - \overline{R}))^4$.

Future research into portfolio optimisation using a four-moment framework may shed further insights into the impacts of skewness and kurtosis on asset allocation as most investors include non-normally distributed asset returns in their portfolios.

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